Engineering Analysis with Boundary Elements 57 (2015) 88–95

Contents lists available at ScienceDirect

Engineering Analysis with Boundary Elements

journal homepage: www.elsevier.com/locate/enganabound

Simulation of elastic wave propagation in layered materials by the method of fundamental solutions

Ji Lin, Wen Chen*, Linlin Sun

College of Mechanics and Materials, Hohai University, Nanjing, PR China

ARTICLE INFO

Article history:
Received 28 March 2014
Received in revised form
21 October 2014
Accepted 10 November 2014
Available online 15 December 2014

Keywords:
Method of fundamental solutions
Domain decomposition method
Layered materials
Elastic wave propagation

ABSTRACT

In this paper, the method of fundamental solutions (MFS) is applied in combination with the domain decomposition method to the simulation of elastic wave propagation in layered materials. The domain of the problem under consideration is decomposed into several sub-domains. In each sub-domain, the solution is approximated separately by the MFS formulation. At the sub-domain interfaces, continuity of the displacement and traction is imposed as the boundary conditions. The validity of this approach is demonstrated through a series of two- and three-dimensional numerical experiments.

1. Introduction

The simulation of elastic wave propagation in layered materials is of interest in many instances. For example, it can be crucial to wave propagation in fiber materials [1,2], and to seismic disturbance of stratified soil [3,4]. Analytical solutions to such problems are rare. For practical problems, numerical methods are required. Numerical simulation techniques have become an indispensable part of the industrial process [5,6]. It is known to all that the element-based methods such as the finite element method (FEM) [7–9] and the boundary element method (BEM) [10–12] are powerful numerical methods for problems in engineering. The FEM discretizes the domain of interest into small elements. Within these elements, the solutions are approximated by shape functions. However, for large numerical models, the FEM requires a prohibitive computational cost for mesh generation. The BEM has long been recognized as an efficient numerical tool thanks to its distinctive feature that only the boundary needs to be modeled. Despite this advantage, the BEM involves mathematically complex and computationally expensive evaluation of singular or hyper-singular integrals.

In order to overcome these disadvantages, meshless methods have been proposed in recent decades [13–23]. The method of fundamental solutions (MFS) [24–28] is a typical type of boundary-type meshless methodology which can be viewed as the indirect BEM. Similar to the BEM, the MFS works when the fundamental solutions of the governing equations are prescribed. The MFS outperforms the standard BEM in terms of integration free, convergence speed, easy-to-use, and meshfree merits. In the MFS, the solution is approximated by the linear combination of fundamental solutions of the governing equations with source points located outside the solution domain so as to overcome the singularity of the fundamental solutions. The unknown coefficients are determined so that the boundary conditions are imposed. The MFS is first used to approximate the solution of homogeneous elliptic-type partial differential equations. Furthermore, it is used for nonhomogeneous problems in combination with the method of particular solution (MPS) [29,30]. The MFS has already been used for the simulation of a variety of physical problems. A survey of the MFS and some related methods can be found in [31,32].

The objective of this paper is to formulate the MFS formulation for the solution of the elastic wave propagation problem in layered materials. The problems under consideration are solved by the domain decomposition method [33–35] in combination with the MFS. The combination of the MFS and the domain decomposition method was applied to several physical problems in the past. Chen et al. applied the MFS to analyze the eigenanalysis of thin membranes with stringers in [36]. Young et al. studied the degenerate seepage flownet problems by the MFS with the domain decomposition method [37]. Alves et al. considered the application of the MFS to solve crack problems with domain decomposition method [38]. In this paper, the layered materials are decomposed into several sub-domains. In each sub-domain, the MFS is used for simulation. At the sub-domain interfaces, continuity of the displacement and the traction is imposed as the boundary conditions. The final system of equation is constituted by assembling algebraic equations discretized in each sub-domain, based on the compatibility of displacement and equilibrium of traction at adjacent interface nodes. Several numerical examples are employed to verify the performance of the MFS approach. And the numerical results show that they agree well with the solutions obtained by the FEM.

* Corresponding author.
E-mail address: chenwen@hhu.edu.cn (W. Chen).

http://dx.doi.org/10.1016/j.enganabound.2014.11.007
0955-7997/

The remaining of this paper is organized as follows. In Section 2, the MFS for the elastic wave propagation problem in a single material is briefly described. Section 3 introduces the key idea of the domain decomposition method in combination with the MFS for the elastic wave propagation problem in layered materials. Numerical results and conclusions are provided in Sections 4 and 5, respectively.

2. The MFS for the elastic wave problem in a single material

This section provides a brief review on the MFS for the elastic wave problem in a single material. In the absence of body force, and assuming the harmonic time dependence e^{jwt}, the governing equation of the elastic wave propagation problem is reduced to

$$\mu\nabla^2\mathbf{u}+(\lambda+\mu)\nabla\nabla\cdot\mathbf{u}+\rho w^2\mathbf{u}=0, \tag{1}$$

where ρ is the mass density, w is the circular frequency, $\mathbf{u}=[u_1\ u_2\ \cdots\ u_d]$ denotes the displacement, d represents the dimension of the problem, $\nabla=[\partial/\partial x_1\ \partial/\partial x_2\ \cdots\ \partial/\partial x_d]$, λ and μ are the Lamé elastic constants which are defined as follows:

$$\lambda=\frac{vE}{(1+v)(1-2v)},\quad \mu=\frac{E}{2(1+v)}, \tag{2}$$

in which E is the modulus of elasticity, and v is Poisson's ratio.

The strain ε_{ik} is related to the displacement gradients by means of

$$\varepsilon_{ik}=\frac{1}{2}\left(\frac{\partial u_i}{\partial x_k}+\frac{\partial u_k}{\partial x_i}\right),\quad i=1\text{ to }d,\ \ k=1\text{ to }d, \tag{3}$$

And the stress σ_{ik} is related to the strain via Hooke's law by

$$\sigma_{ik}=\lambda\delta_{ik}u_{l,l}+2\mu\varepsilon_{ik},\quad i=1\text{ to }d,\ \ k=1\text{ to }d, \tag{4}$$

where δ_{ik} is the Kronecker delta symbol, and $(,l)$ denotes derivative with respect to x_l. The boundary traction t_i is defined in terms of the stress:

$$t_i=\sigma_{ik}n_k,\quad i=1\text{ to }d, \tag{5}$$

where n_k denotes the coordinates of the outward norm to the boundary.

The fundamental solutions of the displacement for two-dimensional problems are

$$G_{ik}(\mathbf{x},\mathbf{s})=A\delta_{ik}-Br_{,i}r_{,k},\quad i=1,2,\ \ k=1,2, \tag{6}$$

where $\mathbf{x}=[x_1,x_2]$ and $\mathbf{s}=[s_1,s_2]$ are the nodes inside the solution domain and source nodes outside the domain, respectively. A and B are

$$A=\frac{1}{2\pi w^2\rho}\left(k_s^2K_0(jk_sr)+\frac{jK_1(jk_pr)k_p}{r}-\frac{jK_1(jk_sr)k_s}{r}\right),$$

$$B=\frac{1}{2\pi w^2\rho}\left(k_s^2K_0(jk_sr)-k_p^2K_0(jk_pr)\right)+\frac{1}{2\pi w^2\rho}\left(2\frac{jK_1(jk_pr)k_p}{r}-2\frac{jK_1(jk_sr)k_s}{r}\right), \tag{7}$$

where K_i ($i=0$ or 1) are the ith order modified Bessel functions, $k_s=w\sqrt{\rho/\mu}$, $k_p=w\sqrt{\rho/(\lambda+2\mu)}$, $r=\sqrt{(x_1-s_1)^2+(x_2-s_2)^2}$, $r_{,i}=(x_i-s_i)/r$, $i=1,2$. Similarly, the fundamental solutions of the traction for two-dimensional problems are

$$T_{ik}=\lambda\left(A'-B'-\frac{B}{r}\right)r_{,k}n_i+\mu\left(A'-\frac{B}{r}\right)(r_{,n}\delta_{ik}+r_{,i}n_k)-\mu\frac{2B}{r}r_{,k}n_i+2\mu\left(-B'+\frac{2B}{r}\right)r_{,i}r_{,k}r_{,n}, \tag{8}$$

where $\{*\}'$ denotes the derivative with respect to r, and $r_{,n}=r_{,1}n_1+r_{,2}n_2$.

The fundamental solutions of the displacement for three-dimensional problems are

$$G_{ik}(\mathbf{x},\mathbf{s})=A\delta_{ik}-Br_{,i}r_{,k},\quad i=1,2,3,\ \ k=1,2,3, \tag{9}$$

where

$$A=\frac{1}{4\pi w^2\rho}\left(k_s^2\frac{\exp(-jk_sr)}{r}-\left(jk_s+\frac{1}{r}\right)\frac{\exp(-jk_sr)}{r^2}\right)+\frac{1}{4\pi w^2\rho}\left(\left(jk_p+\frac{1}{r}\right)\frac{\exp(-jk_pr)}{r^2}\right),$$

$$B=\frac{1}{4\pi w^2\rho}\left(\left(k_s^2-\frac{3}{r^2}-\frac{3jk_s}{r}\right)\frac{\exp(-jk_sr)}{r}\right)-\frac{1}{4\pi w^2\rho}\left(\left(k_p^2-\frac{3}{r^2}-\frac{3jk_p}{r}\right)\frac{\exp(-jk_pr)}{r}\right), \tag{10}$$

in which $k_s=w\sqrt{\rho/\mu}$, $k_p=w\sqrt{\rho/(\lambda+2\mu)}$, $r_{,i}=(x_i-s_i)/r$, $i=1,2,3$, $r=\sqrt{(x_1-s_1)^2+(x_2-s_2)^2+(x_3-s_3)^2}$.

The fundamental solutions of the traction for three-dimensional problems are

$$T_{ik}=\lambda\left(A'-B'-\frac{2B}{r}\right)r_{,k}n_i+\mu\left(A'-\frac{B}{r}\right)(r_{,n}\delta_{ik}+r_{,i}n_k)-\mu\frac{2B}{r}r_{,k}n_i+2\mu\left(-B'+\frac{2B}{r}\right)r_{,i}r_{,k}r_{,n},\quad i=1,2,3,\ \ k=1,2,3, \tag{11}$$

where $\{*\}'$ denotes the derivative with respect to r, and $r_{,n}=r_{,1}n_1+r_{,2}n_2+r_{,3}n_3$.

In the MFS, the solutions of the displacement and traction are approximated by the linear combinations of the fundamental solutions as follows:

$$u_i(\mathbf{x}_m)=\sum_{k=1}^{d}\sum_{n=1}^{N}\alpha_{kn}G_{ik}(\mathbf{x}_m,\mathbf{s}_n),\quad \mathbf{x}_m\in\Gamma_u,\quad i=1\text{ to }d, \tag{12}$$

$$t_i(\mathbf{x}_m)=\sum_{k=1}^{d}\sum_{n=1}^{N}\alpha_{kn}T_{ik}(\mathbf{x}_m,\mathbf{s}_n),\quad \mathbf{x}_m\in\Gamma_t,\quad i=1\text{ to }d, \tag{13}$$

where Γ_u and Γ_t are displacement boundary condition and traction boundary condition, respectively, $\Gamma_u\cup\Gamma_t$ constructs the whole boundary of the domain of the problem, $\mathbf{s}_n$ is the nth source node outside the solution domain, $\{\alpha_{kn}\}$ are the unknown coefficients which can be obtained by imposing the boundary conditions at all collocation nodes $\{\mathbf{x}_m\}$. Much works have been devoted to the choice of optimal fictitious boundary nodes $\mathbf{s}_n$, interested readers may refer to [24,39] and reference therein. In this study, the locations of the fictitious boundary are pre-assigned, taken to be a curve similar to the real physical boundary.

3. The MFS for elastic wave problem in layered materials

In this section, we consider elastic wave problem in layered materials as shown in Fig. 1, as an example. The layered materials are decomposed into several sub-layers. And in each sub-domain, the MFS formulation is used. In the lth layer, the solution is approximated by

$$u_i^l(\mathbf{x}_m)=\sum_{k=1}^{d}\sum_{n=1}^{N}\alpha_{kn}^lG_{ik}(\mathbf{x}_m,\mathbf{s}_n), \tag{14}$$

$$t_i^l(\mathbf{x}_m)=\sum_{k=1}^{d}\sum_{n=1}^{N}\alpha_{kn}^lT_{ik}(\mathbf{x}_m,\mathbf{s}_n), \tag{15}$$

And for $\mathbf{x}_m\in(l+1)$th layer, we have

$$u_i^{l+1}(\mathbf{x}_m)=\sum_{k=1}^{d}\sum_{n=1}^{N}\alpha_{kn}^{l+1}G_{ik}(\mathbf{x}_m,\mathbf{s}_n), \tag{16}$$