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2.5D elastic wave propagation in non-homogeneous media coupling the BEM and MLPG methods

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ABSTRACT

This paper describes a 2.5D numerical frequency domain model based on the mutual coupling of the boundary element method (BEM) and the meshless local Petrov–Galerkin (MLPG) method for simulating elastic wave propagation in non-homogeneous media, when the geometry does not change in the z direction. The BEM is used to model the propagation within the unbounded homogeneous domain while the MLPG is used to simulate the confined non-homogeneous domains. The coupling of the two numerical techniques is accomplished directly at the nodal points located at the common interface. Continuity of mechanical displacements and tractions at the interface is imposed through the collocation of continuity equations on the interface with use of the moving least-squares (MLS) scheme. The MLS was also chosen for the approximation of the trial functions for the MLPG formulation.

The coupled BEM–MLPG approach is verified against the results provided by an analytical solution developed for a circular multi-layered subdomain, in which the elastic material properties within the circular non-homogeneous region are assumed to vary in the radial direction. Finally, an unbounded medium containing two non-homogeneous inclusions excited by a blast load is used to illustrate the applicability of the proposed model.

1. Introduction

In this paper we describe a new approach for the analysis of a classical problem in engineering mechanics of 2.5D elastic wave propagation in non-homogeneous media. Elastic wave propagation in non-homogeneous media is a significant research topic in various fields of engineering and science including geotechnics, earthquake engineering and non-destructive testing.

The analytical solutions to these problems are limited to simple, regular geometries [1–3]. That is why numerical methods are required for general practical problems. Several numerical tools have been developed for elastic wave propagation analysis, including the well-known boundary element method (BEM) [4–6], the finite element method (FEM) [7,8], the hybrid numerical method [9] and meshless methods [10,11]. The numerical meshfree analysis of wave propagation in functionally graded media is set out in [12–15]. Wilcox et al. [16] introduced a higher order discontinuous Galerkin scheme for the 3D wave propagation analysis of coupled elastic–acoustic media that can be applied to seismic exploration and geophysical problems.

Many engineering problems in geotechnics and soil–structure interactions are very complex and can grow to a large scale with high computational demands. In such cases supercomputers can be used [17] or the size of the problem can be reduced with use of infinite elements [18,19]. The BEM is particularly useful for problems of large scale unbounded domains since the far field boundary conditions are automatically satisfied.

Several assumptions are possible that may reduce computational efforts. In certain cases the geometry can be considered longitudinally invariant; this is a valid assumption for roads, railway tracks, tunnels, pipelines, dams and alluvial valleys [20]. A two-and-a-half-dimensional (2.5D) approach can be applied to such problems of longitudinally invariant structures [21]. The Fourier transform of the longitudinal coordinate can then represent the 3D response of the structure in a 2D discretized domain (cross-section). Yang and Hung [22] analyzed visco-elastic bodies subjected to moving loads by means of a 2.5D finite and infinite element approach. A 2.5D BEM approach was also applied to layered elastic and acoustic formations by Tadeu and Antonio [23] and to seismic analyses [24]. An overview of the structural response to moving loads analyzed by finite element and boundary element schemes is given by Andersen et al. [25]. Boundary element methods have been popular for recovering solutions, primarily in the frequency domain. Fast Fourier transform may be then used for the evaluation of time response. Accurate and stable implicit [26] or explicit [27] time integration schemes should be used to analyze wave propagations in the time domain.

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However, the BEM can only be used for analyzing more general geometries and media when the relevant fundamental solutions or Green's functions, required in the boundary integral equation, are known. But for problems involving non-homogenous media, with variation of elastic material properties, the fundamental solution is generally unavailable in the closed form. The BEM also requires the correct integration of the resulting singular and hypersingular integrals to guarantee its efficiency [28]. Mesh based methods such as the FEM also suffer some disadvantages. For example, if the model to be analyzed is too complex and large the mesh generation process, which is characteristic for these methods, becomes very time-consuming and requires considerable computational effort. Allowing the coarse element meshes may restrict the models to low frequencies if we wish to maintain accuracy.

Therefore, in recent years, a different type of numerical method has been developed as an alternative to the well established mesh-based methods or the BEM, known as meshless methods or element free methods.

These methods require neither domain nor boundary discretization and consequently no information on the connectivity between nodal points and elements is needed, which eliminates some of the mathematical complexity of mesh-based methods and provides accurate solutions at substantially lower computational cost. One of the advantages of meshless methods is their ability to efficiently treat problems with continuously non-homogeneous domains, since the unknown field quantities are approximated only in terms of nodes instead of finite elements, thus the continuous variation of material properties is maintained exactly. The same does not occur in case of mesh-based methods such as the FEM, where the material properties are constant for each finite element leading to piecewise homogeneous material properties in the considered domain.

Unlike some of the methods mentioned above, the MLPG method [29,30] is a truly meshless method since it does not need a background mesh for the numerical integration. It is based on the local weak form of governing equations over small subdomains specified for each nodal point. All integrals can be easily evaluated over these regularly shaped, overlapping subdomains of arbitrary shape (in general, circles for 2D problems and spheres for 3D problems) and their respective boundaries. There is only one nodal point in each subdomain, thus the local sense of the approach is kept. In the MLPG method, trial and test functions can be chosen from different functional spaces, which allow several MLPG formulations [31]. The application of the MLPG method to the analysis of a broad range of scientific problems is summarized in the review article by Sladek et al. [32].

However, like mesh-based techniques, the meshless methods have their own disadvantages and limitations. The interpolations and the algorithm implementation tend to be computationally expensive and these methods may not be efficient for problems with infinite and semi-infinite domains [33]. Therefore, many researchers have been proposing the coupling of appropriately selected methods to alleviate specific limitations of individual methods and improve efficiency, accuracy and flexibility. The MLPG method has been coupled with the FEM for problems involving elasticity problems [34], potential problems [35] or electromagnetic field computations [36]. Tadeu et al. [37] used a coupled BEM–MLPG approach for the thermal analysis of non-homogeneous media. Direct coupling with the use of an MLS approximation scheme was employed. A similar technique was also used for the acoustic analysis of non-homogeneous inclusions [38]. Other examples include combinations of the BEM with the method of fundamental solutions (MFS) [39,40], BEM with meshless Kansa's method [41], FEM with EFG method [42,43] and BEM with EFG method [44]. Alves Costa et al. [45] proposed a coupled FEM–BEM approach for the 2.5D analysis of track-ground vibrations. The environmental impact of railway traffic and mitigation of track vibration have been studied and the results compared with experimental measurements. The coupling of the BEM and MFS for the 2.5D analysis of elastic wave propagation in the frequency domain is described in [46]. Accurate 2.5D MFS analyses have also been presented [47] as well as an analytical 2.5D approach for multilayered media [48].

Certain heterogeneous media can be characterized as multi-component composites with smooth variation of the volume fraction of the constituents. If the volume fraction of the constituents predominantly varies in a particular direction, we are talking about functionally graded materials (FGMs). A review of various aspects of FGMs can be found in the monograph by Suresh and Mortensen [49]. Liu et al. [50] presented a comparison of various numerical techniques for the elastodynamic analysis of FGMs. Han et al. [9] analyzed transient elastic waves in FGM plates and an FGM cylinder [51]. As mentioned above, meshless methods are advantageous for the analysis of elastodynamics and elastic wave propagation in continuously non-homogeneous media such as FGMs. Sladek et al. [52] applied the MLPG to elastodynamic problems in continuously non-homogeneous bodies. Efficient analytical evaluation of integrals in the meshless local integral equation method [53–55] was implemented for elastodynamic problems by Soares Jr et al. [56] and by Wen and Aliabadi [57]. The local boundary integrals are obtained in closed form, therefore no domain or boundary integrals have to be calculated numerically. Racz and Bui [58] introduced a novel adaptive integration technique for meshless methods. The elastic wave propagation in a functionally graded nanocomposite reinforced by carbon nanotubes has recently been analyzed by means of the MLPG [59].

In this paper we propose applying a BEM and MLPG coupling formulated in the frequency domain to the analysis of elastic wave propagation through a 2.5D unbounded homogeneous domain containing inclusions with a non-homogeneous variation of elastic properties. The elastic material properties inside the inclusion are assumed to vary in a smooth fashion. The advantages of each method are exploited by using the BEM for the homogeneous unbounded domain and the MLPG for the non-homogeneous inclusion. Nodal points are introduced inside the non-homogeneous domain and on the interface, where the same nodal points are used for the specification of boundary elements. The continuity conditions for the displacements and tractions are specified at these interface nodes. The moving-least squares (MLS) approximation is applied to the MLPG formulation for the approximation of unknown nodal quantities inside the non-homogeneous domain, and to the continuity conditions. This direct coupling method does not require the iterative technique or the concept of overlapping ‘double nodes’ for mutual BEM–MLPG coupling.

In the case of 2.5D wave propagation in elastic media presented here, the resulting hypersingular kernels appearing in the BEM formulation can be computed analytically [60].

The proposed coupled BEM–MLPG approach is verified against an analytical solution known for having a simple geometry. Circular cylindrical domains are modeled to illustrate the efficiency of the proposed methodology, since in this case analytical solutions can be defined. Finally, a numerical example is used to illustrate the applicability of the proposed method. The responses in the time domain are obtained by means of a fast inverse Fourier transform. Some conclusions are drawn and the quality of the numerical results is discussed.

2. Definition of the problem

Elastic wave propagation in a non-homogeneous isotropic medium is governed by the following equilibrium equation:

$$\sigma_{ij,j}(\mathbf{x},t) = \rho \ddot{u}_i(\mathbf{x},t) \quad (1)$$

where σ_{ij} is the stress tensor, u_i are mechanical displacements and ρ is the mass density. A comma followed by an index denotes