Engineering Analysis with Boundary Elements 61 (2015) 104–113

Contents lists available at ScienceDirect

Engineering Analysis with Boundary Elements

journal homepage: www.elsevier.com/locate/enganabound

A local meshless collocation method for solving Landau–Lifschitz–Gilbert equation

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ARTICLE INFO

Article history:
Received 1 May 2015
Received in revised form
14 July 2015
Accepted 15 July 2015
Available online 4 August 2015

Keywords:
Radial basis functions
Landau–Lifschitz equation
Landau–Lifschitz–Gilbert equation
Local meshless methods
Finite collocation method
Finite differences

ABSTRACT

This paper is concerned with a meshless simulation of the two dimensional Landau–Lifschitz–Gilbert (LLG) equation which describes the dynamics of the magnetization inside a ferromagnetic body. After elimination of the time variable by a suitable finite difference scheme, a combination of the meshless local RBF and the finite collocation method is used for spatial discretizations of the field variables. Three test problems are numerically investigated and the results reveal the effectiveness of the method.

1. Introduction

Ferromagnetic materials have long been the subject of scientific studies. It is well known that they play an important role in the industry such as magnetic sensors, actuators, reading-writing heads, information storage media, passive circuit elements [1]. The first description of the evolution of the magnetization inside a ferromagnetic material was suggested by Landau and Lifschitz in 1935 [2]. They proposed the following equation:

$$\frac{\partial \mathbf{m}(\mathbf{x},t)}{\partial t} = \alpha_1 \mathbf{m}(\mathbf{x},t) \times \mathbf{H}_{eff} - \alpha_2 \mathbf{m}(\mathbf{x},t) \times (\mathbf{m}(\mathbf{x},t) \times \mathbf{H}_{eff}), \quad \mathbf{x} \in \Omega, \ 0 < t < T \tag{1.1}$$

with the given initial condition

$$\mathbf{m}(\mathbf{x},0) = \mathbf{m}_0(\mathbf{x}), \tag{1.2}$$

and the homogeneous Neumann boundary condition

$$\frac{\partial \mathbf{m}(\mathbf{x})}{\partial \mathbf{n}} = 0, \quad \mathbf{x} \in \partial\Omega, \tag{1.3}$$

where $\Omega \subset \mathbb{R}^2$ is a multi-connected bounded domain, $\mathbf{m} : (0,T) \times \Omega \to \mathbb{S}^2$ is an unknown magnetization vector field and '$\times$' is the three dimensional cross product. Also $\mathbf{n}$ is the unit outward normal vector on Ω and $\mathbb{S}^2 = \{\mathbf{x} \in \mathbb{R}^3 : |\mathbf{x}| = 1\}$ is the unit sphere. It means that $\mathbf{m}$ has a length-preserving property during the evolution process. This comes from a scalar multiplication of (1.1) with $\mathbf{m}$. So $\mathbf{m}_t \cdot \mathbf{m} = 0$; then, $\frac{\partial |\mathbf{m}|^2}{\partial t} = 0$, which implies $|\mathbf{m}(\mathbf{x},t)|$ is constant for all t and each $\mathbf{x}$ as it is the case for $|\mathbf{m}(\mathbf{x},0)|$. Also, $\alpha_1 \neq 0$ is a gyromagnetic constant factor, and $\alpha_2 > 0$ is a damping constant parameter which makes the equation parabolic. Here, $\mathbf{H}_{eff}$ is the effective field and is the (opposite of the) functional derivative of the free energy E

$$\mathbf{H}_{eff} = -\frac{\partial E}{\partial \mathbf{m}}.$$

In the simplest situation when the energy functional consists of the exchange energy only, the effective field is $\mathbf{H}_{eff}$, $\Delta\mathbf{m}$, and energy is

$$E(\mathbf{m}) = \frac{1}{2}\int_\Omega |\nabla \mathbf{m}|^2 \, d\Omega.$$

It is easy to check that

$$\frac{d}{dt}E(\mathbf{m}) = -\alpha_2 \int_\Omega |\mathbf{m} \times \Delta\mathbf{m}|^2 \, d\Omega = -\frac{\alpha_2}{1+\alpha_2^2}\int_\Omega |\partial_t \mathbf{m}|^2 \, d\Omega.$$

That is, this problem has energy dissipation property for the case $\alpha_2 > 0$ and energy conservation property for the case $\alpha_2 = 0$. Two special cases of (1.1) are of particular geometrical interest. If $\alpha_2 = 0$, then Eq. (1.1) reduces to the gyromagnetic term

$$\mathbf{m}_t = \alpha_1 \mathbf{m} \times \Delta\mathbf{m}. \tag{1.4}$$

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http://dx.doi.org/10.1016/j.enganabound.2015.07.010
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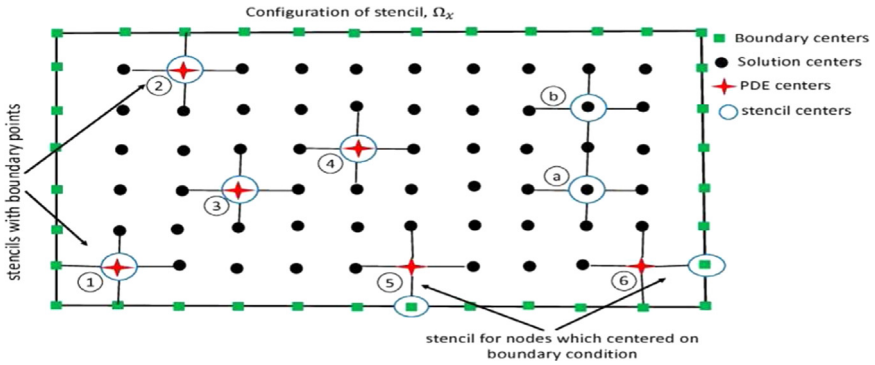

Fig. 1. Configuration of stencils.

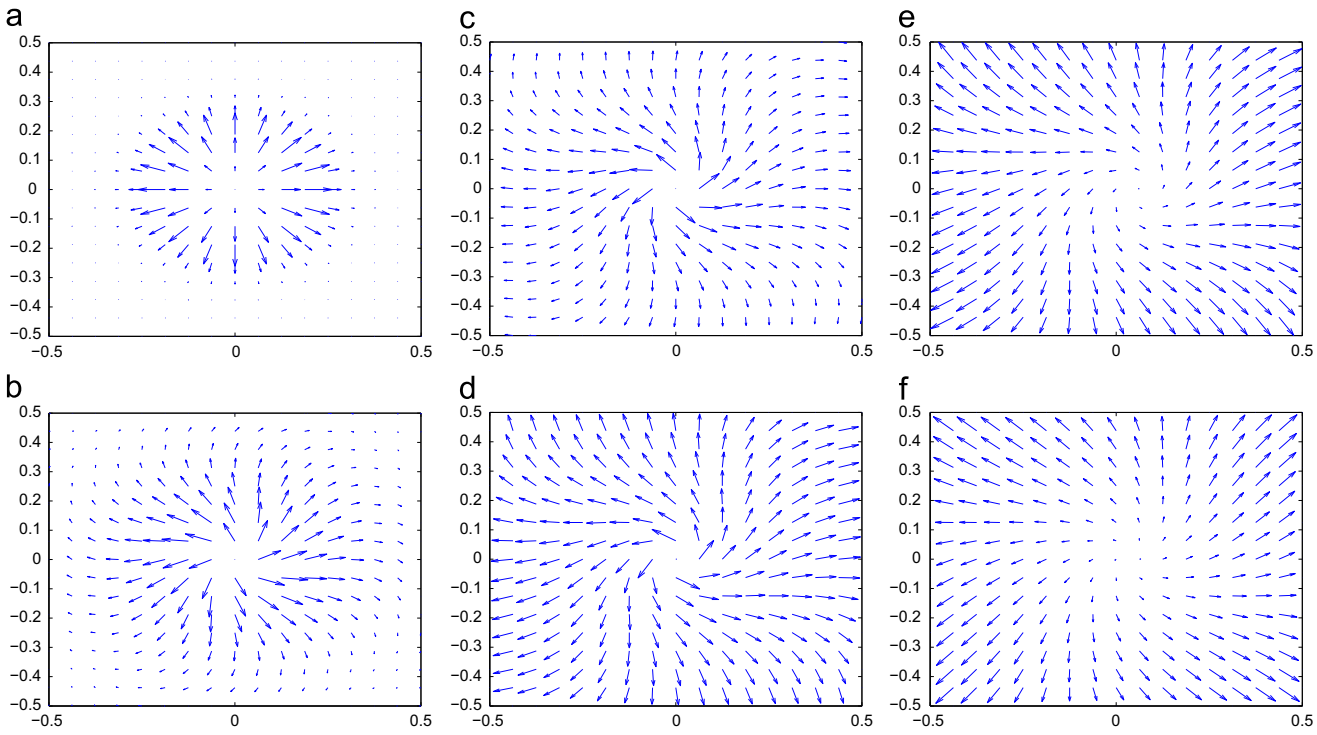

Fig. 2. Numerical approximation of $\dot{\mathbf{m}}(x,y,t)$ for Ex. 1 with $N_1 = 17^2$, $\alpha_2 = 1$, $s=1$ and $c=2$: (a) $t=0$, (b) $t=0.0119$, (c) $t=0.0295$, (d) $t=0.0495$, (e) $t=0.0588$ and (f) $t=0.0646$.

Solutions of (1.4) are often called Schrödinger maps, because the equation is of the type of a nonlinear Schrödinger equation (which is most obvious when $\mathbf{m}$ is composed with the stereographic projection) [3]. This equation describes the Hamiltonian (or symplectic) flow of harmonic maps to $\mathbb{S}^2$ [3]. If $\alpha_2 = \infty$, then Eq. (1.1) reduces to

$$\mathbf{m}_t = -\mathbf{m} \times (\mathbf{m} \times \Delta \mathbf{m});$$
$$\mathbf{m}_t = \Delta \mathbf{m} + |\nabla \mathbf{m}|^2 \mathbf{m};$$

where we have used the fact that $(\mathbf{m}, \mathbf{m}) = 1$. This equation describes the heat flow of harmonic maps.

In [4], Gilbert introduces a different approach for description of damped precession. He introduced the following equation:

$$\alpha_1 \mathbf{m}_t + \alpha_2 \mathbf{m} \times \mathbf{m}_t = (\alpha_1^2 + \alpha_2^2)\mathbf{m} \times \Delta \mathbf{m}, \tag{1.5}$$

which was named as the Landau–Lifshitz–Gilbert (LLG) equation. Eqs. (1.1) and (1.5) are mathematically equivalent and its proof can be found in [5]. Although Eqs. (1.1) and (1.5) are mathematically equivalent; but, (1.5) gives more numerically stable solutions than (1.1) because the latter has a double cross term, namely $\mathbf{m} \times (\mathbf{m} \times \mathbf{H}_{eff})$. So instead of Eqs. (1.1)–(1.3) , we solve (1.5) with the initial condition (1.2) and boundary condition (1.3). LLG equation is a nonlinear system of partial differential equations. Due to the nonlinearity that appears in the LLG equation, finding numerical approximations of Eq. (1.5) is very important for applications, and numerous strategies already exist in the literature [1,3,5,25]. Classical schemes are based on finite differences that are, as usual, well adapted to Cartesian grids. On the other hand, finite element approximation is other method that applied in [6,25]. This paper aims to numerically simulate Eq. (1.5) via a local meshless method. Meshless methods [7–11,15–20] are very attractive and effective for solving boundary value problems, because they involve simple preprocessing, arbitrary node distribution and flexibility of placing nodes at arbitrary locations, straightforward adaptive refinement, versatility in solving large deformation and also have the high order continuity and the ability to treat the evolution of non-smooth solutions, which is very useful to solve PDE problems. Many of them are derived from a weak-form formulation on a global domain or a set of local