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Free vibration of moderately thick functionally graded plates by a meshless local natural neighbor interpolation method

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ABSTRACT

Using a meshless local natural neighbor interpolation (MLNNI) method, natural frequencies of moderately thick plates made of functionally graded materials (FGMs) are analyzed in this paper based on the first-order shear deformation theory (FSDT), which is employed to take into account the transverse shear strain and rotary inertia. The material properties of the plates are assumed to vary across the thickness direction by a simple power rule of the volume fractions of the constituents. In the present method, a set of distinct nodes are randomly distributed over the middle plane of the considered plate and each node is surrounded by a polygonal sub-domain. The trial functions are constructed by the natural neighbor interpolation, which makes the constructed shape functions possess Kronecker delta property and thus no special techniques are required to enforce the essential boundary conditions. The order of integrands involved in domain integrals is reduced due to the use of three-node triangular FEM shape functions as test functions. The natural frequencies computed by the present method are found to agree well with those reported in the literature, which demonstrates the versatility of the present method for free vibration analysis of moderately thick functionally graded plates.

1. Introduction

The concepts of functionally graded materials (FGMs) were proposed by material scientists in the Sendai area of Japan [1] in 1984 and then developed rapidly over the world. In FGMs, material properties are inhomogeneous and changing gradually and smoothly in position so that no distinct physical interface appears. The absence of distinct interface avoids stress concentration, which may occur at the interface of two bonded dissimilar materials when heated or cooled. With the advantageous features in many practical applications, FGMs have been extensively studied over the world by scientists [2–7].

Functionally graded (FG) plates are usually designed purposefully to resist high thermal flux and corrosion by a ceramic-rich surface and to sustain mechanical loads by another metal-rich surface, possessing continuously changed effective material properties in the thickness direction only. The analyses of FG plates are more and more attractive due to their increasing applications. A few researchers [5,6] employed classical plate theory to analyze vibration and static behavior of thin FG plates. However, the classical plate theory under predicts deflections and over predicts frequencies as well as buckling loads for moderately thick plates because it does not take into account the transverse shear deformation effect [7]. Consequently, many shear deformation theories accounting for transverse shear effects have been developed to overcome the deficiencies of the classical plate theory. The first order shear deformation theory (FSDT) based on Reissner [8] and Mindlin [9] accounted for the transverse shear effects by means of linear variation of in-plane displacements across the thickness. Due to its high efficiency and simplicity, the FSDT is considered to be a pioneering theory and has been widely used for analyzing moderately thick FG plates [10–20]. Unfortunately, a shear correction factor is required in the FSDT to amend the effect of uniform transverse stress in shear forces. The shear correction factor is hard to determine since it depends on many parameters. To avoid the use of shear correction factor, some higher-order shear deformation theories (HSDTs) [21–25] have been proposed. Although the HSDTs do not require the shear correction factor, their equations of motion are more complicated than those of the FSDT.

Although several analytical solutions [26,27] have been presented for the analysis of FG plates, it is in general difficult to obtain the exact solution for all problems because of the complexity of mathematics. Therefore numerical methods are essential for simulating FG plates. Among them the finite element method (FEM) has attracted considerable attention due to its accuracy,

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convenience and flexibility. However in the case of complex geometries the generation of highly distorted meshes is common. The distortion of meshes causes low quality shape functions which can affect the performance of the method. So far Long and Cen's research group has proposed various quadrilateral area coordinate (QAC) elements [28–30], which successfully avoid the loss of accuracy when element shapes are distorted. Besides, more and more attention is paid to the development and application of meshless methods [31–36], which construct the approximation solutions completely in terms of a set of orderly or scattered nodes that discretize the problem domain, which means not only that the burdensome work of mesh generation is avoided, but also more accurate description of irregular complex geometries can be achieved. The success of the meshless methods has been reported in solving a wide range of computational problems, proving popular due to their rapid convergence characteristics and their ability to obtain highly accurate solutions for problems involving stress discontinuities. As a result, the application of the meshless methods in analyzing FG plates is of great interest and deserves study. Recently, some meshless approaches [10–20,23,24] were devised for the analysis of FG plates. In addition, Liew et al. [37] published a review of meshless methods for the analyses of laminated and functionally graded plates and shells.

In this paper, the meshless local natural neighbor interpolation (MLNNI) method [38,39] is further formulated for free vibration analysis of moderately thick FG plates. In the MLNNI method, the test and trial functions are chosen from different functional spaces, with trial functions being interpolated by the natural neighbor interpolation (NNI) [40], and the three-node triangular FEM shape functions used as the test functions. Therefore the MLNNI method combines the advantage of easy imposition of essential boundary conditions of the NNI with some prominent features of the meshless local Petrov–Galerkin (MLPG) method. These salient advantages of the MLNNI method have been further demonstrated by some recent developments [41–45]. The first-order shear deformation theory is used to take into account the transverse shear strain and rotary inertia. The elastic properties of the FG plates are determined by the volume fractions of their constituents, which vary continuously through their thickness according to a power law. The local weak forms of governing equations are established based on local polygonal sub-domains centered at each node. The numerical studies are conducted to demonstrate the accuracy, stability and effectiveness of the present method.

2. Functionally graded material properties

As shown in Fig. 1, a functionally graded (FG) plate composed of ceramic and metal phases is considered. Without losing generality, it is assumed that the top surface of a FG plate is ceramic rich and bottom is metal rich. The material properties vary across the

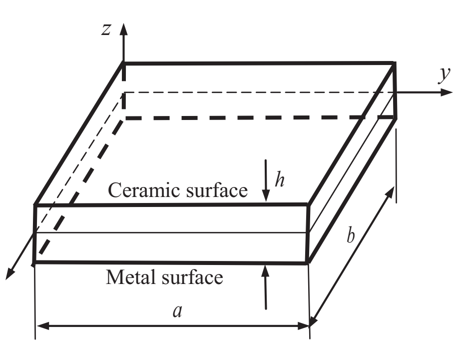

Fig. 1. Sketch of functionally graded plate.

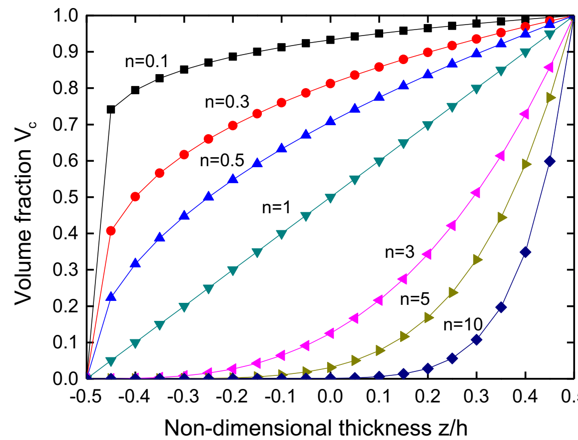

Fig. 2. Volume fraction versus thickness.

thickness according to the following equation:

$$P(z) = P_c V_c + P_m V_m \tag{1}$$

where P represents the effective material properties including Young's modulus E, density ρ, and Poisson's ratio ν. P_c, V_c and P_m, V_m are the material properties and volume fractions of the ceramic and metallic constituent materials, respectively. The relation between V_c and V_m can be given as follows:

$$V_c + V_m = 1 \tag{2}$$

In the present study, a power law variation of the material properties is considered:

$$V_c = \left(\frac{1}{2} + \frac{z}{h}\right)^n, \quad V_m = 1 - \left(\frac{1}{2} + \frac{z}{h}\right)^n \quad (n \geq 0) \tag{3}$$

where h represents the thickness of the plate and n the volume fraction exponent. Fig. 2 shows the variation of the volume fraction through the thickness for different exponents n. It can be assumed that the material is isotropic within the plane parallel to the middle plane of the plate. If $n = 0$ the plate is made of full ceramic, while for n approaching infinity the case of the fully metallic plate is obtained.

3. Equations of motion for moderately thick FG plates

According to the FSDT [10–20], the displacement field can be expressed as

$$u(x,y,z,t) = u_0(x,y,t) + z\theta_x(x,y,t) \tag{4a}$$

$$v(x,y,z,t) = v_0(x,y,t) + z\theta_y(x,y,t) \tag{4b}$$

$$w(x,y,z,t) = w_0(x,y,t) \tag{4c}$$

where u_0, v_0 and w_0 denote the displacements of the mid-plane of the plate in the x, y, and z directions, and θ_x and θ_y represent the rotations of a transverse normal about positive y and negative x axes, respectively. Clearly, a three-dimensional problem can be reduced to a pseudo two-dimensional problem with the assumption of the FSDT.

The linear strain–displacement relations based on the FSDT are expressed in a vector form as

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} \frac{\partial u_0}{\partial x} \\ \frac{\partial v_0}{\partial y} \\ \frac{\partial u_0}{\partial y} + \frac{\partial v_0}{\partial x} \end{Bmatrix} + z \begin{Bmatrix} \frac{\partial \theta_x}{\partial x} \\ \frac{\partial \theta_y}{\partial y} \\ \frac{\partial \theta_x}{\partial y} + \frac{\partial \theta_y}{\partial x} \end{Bmatrix} = \boldsymbol{\varepsilon}^0 + z\boldsymbol{\kappa} \tag{5a}$$