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A new BEM for solving 2D and 3D elastoplastic problems without initial stresses/strains

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ABSTRACT

In this paper, new boundary-domain integral equations are derived for solving two- and three-dimensional elastoplastic problems. In the derived formulations, domain integrals associated with initial stresses (strains) are avoided to use, and material nonlinearities are implicitly embodied in the integrand kernels associated with the constitutive tensor. As a result, only displacements and tractions are explicitly involved in the ultimate integral equations which are easily solved by employing a mature efficient non-linear equation solver. When materials yield in response to applied forces, the constitutive tensor (slope of the stress–strain curve for a uniaxial stress state) becomes discontinuous between the elastic and plastic states, and the effect of this non-homogeneity of constitutive tensor is embodied by an additional interface integral appearing in the integral equations which include the differences of elastic and plastic constitutive tensors. The domain is discretized into internal cells to evaluate the resulted domain integrals. An incremental variable stiffness iterative algorithm is developed for solving the system of equations. Numerical examples are given to verify the correctness of the proposed BEM formulations.

1. Introduction

The earliest work on elastoplastic boundary element method was done by Swedlow and Cruse [1], who presented the elastoplastic formulations based on an extended form of Somigliana's identity by incorporating a volume integral involving plastic strains (the initial strains). Then two- and three-dimensional elastoplastic algorithms were implemented [2–4]. However, it emerged that strongly singular nature of the volume integral involving initial strains in the interior stress equations had been overlooked. The corrected formulations were published by Mukherjee [5], Bui [6], Telles and Brebbia [7]. By using a similar initial strain formulation, Kumar and Mukherjee [8] implemented the first viscoplastic analyses. The first initial stress formulation in the boundary element method was developed by Banerjee and Cathie [9]. Later, an advanced formulation of the boundary element method was developed by Raveendra [10], Banerjee et al. [11], Banerjee and Raveendra [12] for inelastic analysis based on the earlier initial stress approach.

One important task in nonlinear BEM is the solution strategy to the system of equations. Several algorithms using domain discretization methods have been developed. More often than not the algorithms are 'explicit' methods as described in detail by Telles [13] and Banerjee [14]. These solution algorithms can be roughly divided into two groups, i.e., the initial strain approach [2,3,7,8,15–17] and the initial stress approach [9,10,18–23].

The convergence of 'explicit' methods is slow, so implicit solution schemes were investigated, on account for their unconditional stability [24–28]. Among these works, Bonnet and Mukherjee [24], Bonnet et al. [29] first applied the consistent tangent operator method to the boundary element method. This method, which was proposed by Simo and Taylor [30] in the finite element method context, exploits the quadratic rate of convergence which can be achieved by utilizing consistent elastoplastic constitutive relations in the Newton–Raphson iterative process. Dong [31] extended the consistent tangent operator method to axisymmetrically elastoplastic problems. Although this iterative method is relatively easy to code, it requires considerable computer memory. This is because the system of equations is formulated in terms of strain increments which have six degrees of freedom at each node for 3D problems.

A different type of solution strategy (incremental variable stiffness) has been successfully demonstrated by Banerjee and his co-workers [10,14,21,32,33]. In this scheme, the internal variables are eliminated, by expressing them in terms of boundary variables, and consequently no iteration is needed if small increments are used. Based on this formulation, Chopra and Dargush [20] described a Newton–Raphson solution algorithm for solving

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the non-linear system of equations, in which both the boundary unknowns and plastic multipliers were used as primary unknowns of the system of equations. Gao and Davies [26] proposed a novel incremental variable stiffness iterative algorithm based on the Newton–Raphson iterative scheme. In this algorithm, the plastic multipliers (only) are used as the primary unknowns, with the advantage that the number of degrees of freedom is equal to the number of yield nodes in the current increment. Consequently, both computational time and computer memory can be substantially reduced. Based on this method, a self-contained Fortran code for solving elastoplastic problems was published in the book [34], which is the first elastoplastic code entering the public area.

Deng [35] presented a nonlinear complementarity approach to solve elastoplastic problems. In the application of rolling engineering analysis, Xiao [36,37] applied boundary element method with initial stress to evaluate three dimensional frictional contact problems, and elastoplastic material behavior is taken into account by means of the initial stress formulation. By applying fast multipole method (FMM) to BEM, Wang and Yao [38] proposed a fast multipole boundary element method for the analysis of two-dimensional elastoplastic problems.

It is worth mentioning that the previous boundary element formulations for solving elastoplasticity problems were all based on initial stress or initial strain approaches. The existence of domain integrals associated with unknown initial stresses (strains) makes it difficult to formulate system of equations with fewer unknown variables. Especially, in solving multi-medium elastoplastic problems, the widely used multi-domain boundary element method (MDBEM) [39] is very difficult to be directly implemented based on the existing initial stress (strain) integral equations. Also it is difficult to couple with other numerical methods.

In this paper, a new method for solving elastoplastic problems using BEM is developed based on a combination of source point isolation technique [40] and interface integral method [41,42]. The feature of the method is that no initial stresses or initial strains are explicitly appeared in the integral equations and only displacements and tractions are explicitly involved as the basic physical variables. Material nonlinearities are implicitly included in the integral kernels associated with the incremental elastoplastic constitutive tensor. Comparing to the conventional methods, the proposed method has the advantages of convenient to solve multi-medium elastoplastic problems and easy to couple with other numerical methods. A novel effective incremental variable stiffness interactive algorithm is developed for solving the non-linear system of equations based on the Newton–Raphson iterative scheme. Numerical examples are given to verify the correctness of the proposed BEM formulations.

2. Constitutive relations of rate-independent elastoplasticity

Unlike the linear case, we must deal with incremental quantities, denoted by the superposed period. In elastoplastic deformation, total strain increments can be decomposed into elastic and plastic parts as follows:

$$\dot{\varepsilon}_{kl} = \dot{\varepsilon}^e_{kl} + \dot{\varepsilon}^p_{kl} \tag{1}$$

The incremental stress–strain response can then be written in the form

$$\dot{\sigma}_{ij} = D^e_{ijkl}\dot{\varepsilon}^e_{kl} = D^e_{ijkl}(\dot{\varepsilon}_{kl} - \dot{\varepsilon}^p_{kl}) \tag{2}$$

where $\dot{\varepsilon}_{kl}$ is the total strain increment; $\dot{\varepsilon}^p_{kl}$ is the plastic strain increment; and D^e_{ijkl} is the fourth order elastic constitutive tensor with the following form:

$$D^e_{ijkl} = G\left(\frac{2\nu}{1-2\nu}\delta_{ij}\delta_{kl} + \delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk}\right) \tag{3}$$

in which G is the shear modulus; ν is Poisson's ratio and δ_{ij} represents the Kronecker delta.

In the classical (flow) theory of plasticity, the general elastoplastic constitutive relations are based on Drucker's postulate, e.g. [43,44]. However, Drucker's postulate is only suitable for stable materials under stress control with perfect plasticity as a limiting case [45]. In this paper, the elastoplastic constitutive relationships based on Il'iushin's postulate [46] are adopted, which is suitable for both stable and unstable behaviour. For simplicity, the following yield function is considered:

$$f(\sigma_{ij}, \bar{\varepsilon}^p) = \bar{f}(\sigma_{ij}) - k(\bar{\varepsilon}^p) = 0 \tag{4}$$

where $f(\sigma_{ij}, \bar{\varepsilon}^p)$ is the yield function determined by current state of stress and plastic deformation history; $\bar{\varepsilon}^p$ is the cumulated equivalent plastic strain; $\bar{f}(\sigma_{ij})$ is a function of current state of stress, the equivalent uniaxial stress; and $k(\bar{\varepsilon}^p)$ is the uniaxial yield stress. Concrete forms of $\bar{\varepsilon}^p$, $k(\bar{\varepsilon}^p)$ and $\bar{f}(\sigma_{ij})$ can be found in Ref. [34].

The Il'iushin's postulate states that the work done in a closed strain cycle is non-negative and yields the following results [47]:

$$\dot{\varepsilon}^p_{ij} = \dot{\lambda}\frac{\partial f}{\partial \sigma_{ij}} \tag{5}$$

where $\dot{\lambda}$ is the non-negative plastic multiplier increment.

Making the use of consistency condition and hardening rules, the expression for the plastic multiplier can be derived from Eqs. (1) to (5):

$$\dot{\lambda} = \frac{1}{\psi}\frac{\partial f}{\partial \sigma_{ij}}\dot{\sigma}^e_{ij} \tag{6}$$

Then, substituting Eqs. (6) into (5), and the result into Eq. (2), the elastoplastic constitutive stress–strain relation can be derived as

$$\dot{\sigma}_{ij} = D^{ep}_{ijkl}\dot{\varepsilon}_{kl} \tag{7}$$

where $\dot{\sigma}_{ij}$ is the real stress increment, $\dot{\varepsilon}_{kl}$ is the total strain increment, and D^{ep}_{ijkl} is the tangent constitutive tensor with the following form [34]:

$$D^{ep}_{ijkl} = D^e_{ijkl} - \frac{1}{\psi}D^e_{ijmn}\frac{\partial f}{\partial \sigma_{mn}}\frac{\partial f}{\partial \sigma_{pq}}D^e_{pqkl} \tag{8}$$

where

$$\psi = \frac{\partial f}{\partial \sigma_{kl}}D^e_{kljs}\frac{\partial f}{\partial \sigma_{js}} + Hh_{\varepsilon,\lambda} \tag{9}$$

in which $h_{\varepsilon,\lambda}$ is the partial derivative of internal variable h_ε with respect to plastic multiplier λ, which has the following form:

$$h_{\varepsilon,\lambda} = c'\sqrt{\frac{\partial f}{\partial \sigma_{ij}}\frac{\partial f}{\partial \sigma_{ij}}} \tag{10}$$

where c' is a magnitude of plastic strain increment.

If we set

$$d^f_{ij} = D^e_{ijkl}\frac{\partial f}{\partial \sigma_{kl}} \tag{11}$$

Eq. (8) can be rewritten as follows:

$$D^{ep}_{ijkl} = D^e_{ijkl} - D^p_{ijkl} \tag{12}$$

where

$$D^p_{kljs} = \frac{1}{\psi}d^f_{kl}d^f_{js} \tag{13}$$

In Eqs. (8)–(13), H is the local slope of the uniaxial stress–plastic strain curve, which can be determined experimentally;