Engineering Analysis with Boundary Elements 61 (2015) 282–286

Contents lists available at ScienceDirect

Engineering Analysis with Boundary Elements

journal homepage: www.elsevier.com/locate/enganabound

An efficient boundary integral equation method for multi-frequency acoustics analysis

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ARTICLE INFO

Article history:
Received 21 April 2015
Received in revised form
16 July 2015
Accepted 18 August 2015
Available online 3 September 2015

Keywords:
Boundary integral equation method
Acoustics problems
Multi-frequency calculation
Least square approximate

ABSTRACT

In this paper, a multi-frequency calculation technique based on least square approximate is introduced into the boundary integral equation method (BIEM) for 3D acoustics problems. The quadrilateral constant elements are used in multi-frequency calculation technique. In this method, the exponential term is expanded only when the source point and the field point locate in the same element. Thus, all the diagonal entries in system matrices are independent of the wave number. As a result, the integrals for the diagonal entries in all the final matrices (different frequencies) only are calculated once. Comparing with the original BIEM, the storage requirement for the presented method only adds O(n) (n is the total number of the elements). In addition, the presented method can be used to deal with the full frequency acoustic problems. Numerical examples show the accuracy and efficiency of the presented technique.

1. Introduction

The boundary integral equation method (BIEM) has the advantages comparing with finite element method (FEM) due to the semi-analytic nature, the reductions of the dimension, and the incomparable superiority for solving infinite problems. Hence, it has been a very powerful numerical technique for exterior acoustics problems.

For the acoustic problems governed by Helmholtz equation, the first effort of using the integral equation was made by Jaswon and Symm [1]. Chen and Schweikert [2] solved 3-D sound radiation problems by using Fredholm integral equation of the second kind. Chertock [3] predicted sound radiation from vibrating surfaces using integral equation. However, there is a drawback that only using conventional boundary integral equation method (CBIE) formulation can not get unique solution for the exterior acoustic problems governed by the Helmholtz equation at the eigenfrequencies which are associated with the interior problems [4,5]. These eigenfrequencies which are called fictitious eigenfrequencies has no physical significance for the exterior problems under investigation. In order to deal with this defect, the combined Helmholtz integral equation formulation (CHIEF) is proposed by Schenck [4]. In the method, some additional Helmholtz integral relations were added in the interior domain. This additional relation leads to an over-determined system of equations, which can be solved using a least-squares technique. CHIEF has been widely used for acoustic scattering and radiation problems. Furthermore, lots of improvements have been made by several researchers [6–9].

Due to the frequency dependent character of the coefficient matrices in the BIEM, all the components in the coefficient matrices should be calculated repeatedly for different frequencies, the calculation process would be very time consuming if there are large amounts of frequency steps. Seybert [10] had found that it take about 100 min to compute the surface acoustical pressure of a pulsating sphere in the range of $ka \le 10$ (k is the wave number, a is the sphere radio). Kirkup [11] had shown that it cost more than 100 h to determine the sound radiation of a shaft box of $0.25\ \text{m} \times 0.4\ \text{m} \times 0.6\ \text{m}$ in the range of 400–2400 Hz. Many techniques have been explored to solve the multi-frequency acoustic problems. The frequency interpolation technique was proposed in [12,13]. Wu [14] presented a Green's function interpolation technique. The frequency interpolated transfer function was proposed by Estorff [15]. The matrix interpolation and solution iteration process was developed in [16]. The frequency response function approximation was proposed in [17]. The sine and cosine functions, which are included in the complex exponential function, are approximated by polynomials in [18,19].

Although those methods [18,19] can deal with the multi-frequency acoustic problems, the quantity of the required storage is several times of that in the original BIEM. And the frequencies which can be calculated are limited to the low frequencies in

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http://dx.doi.org/10.1016/j.enganabound.2015.08.006
0955-7997/

[18,19]. To avoid these faults, a new technique is introduced in the BIEM to analyze the 3D multi-frequency acoustic problems in this paper. The CHIEF is employed to avoid non-unique solution in BIEM for exterior acoustic problems in this paper. The exponential terms are approximated by a quadratic polynomial, which is obtained by least-squares approach. In our method, the diagonal entries in the final global coefficient matrices at any frequency can be simply formed by a summation of the frequency independent matrices. The integral computation of other entries can be carried out with the 1 point Gauss integral. The storage requirement only adds O(n). The computational effort spent on the singular and near-singular integration is saved. Therefore, the onerous numerical integration is not involved. At the same time, to meet the requirement of the precision in BIEM for acoustic problems, the maximal element size should satisfy: Size $< \lambda/6$. Once a maximal element size l and some frequency $f0$ meet this requirement, all the acoustic problems with frequency below $f0$ can be evaluated by this method.

This paper is organized as follows. Section 2 mainly reviews the BIEM for acoustic problems. In Section 3, the multi-frequency calculation technique is given followed by numerical examples in Section 4. The paper ends with conclusions and discussions on future work in Section 5.

2. Review of the BIEM

In 2D or 3D spaces, the governing equation for acoustic wave problem is the Helmholtz equation which can be written as:

$$\nabla^2\phi(x) + k^2\phi(x) = 0, \quad x \in E \tag{1}$$

in which x is the field point, E is the acoustic domain, $\phi(x)$ denotes the total sound pressure at x, ∇^2denotes the Laplace operator, $k = 2\pi f/c$ denotes the wave number, f is the frequency, c is the speed of sound in the acoustic medium.

The boundary conditions for the governing equation of acoustic wave problems can be described as:

$$\begin{cases} \text{Dirichlet type} \quad \varphi = \bar{\varphi}, \quad \forall\, x \in S \\ \text{Neumann type} \quad q = \dfrac{\partial\varphi}{\partial n} = \bar{q} = ikc\rho v_n, \quad \forall\, x \in S \\ \text{Impedance type} \quad \varphi = Zv_n, \quad \forall\, x \in S \end{cases}, \tag{2}$$

ρ is the mass density. v_n is the normal velocity. n is the outward normal. Z denotes the specific acoustic impedance. The quantities with over bars indicate given values. $i = \sqrt{-1}$. S is the boundary of the domain. For the exterior acoustic problem, the Sommerfeld radiation condition must be satisfied at infinite field. It is:

$$\lim_{R->\infty}\left[R\left|\frac{\partial\varphi}{\partial R} - ik\varphi\right|\right] = 0 \tag{3}$$

where R is the distance from a fixed origin to a general field point. ϕ is the radiated wave in a radiation problem or the scattered wave in a scattering problem.

The integral representation of the Helmholtz equation is:

$$c(P_0)\phi(P_0) = \int_S G(P_0, P)q(P)dS(P) - \int_S \frac{\partial G(P_0, P)}{\partial n}\phi(P)dS(P) + \phi^I(P_0), \tag{4}$$

here $G(P_0, P) = \frac{e^{ikr}}{4\pi r}$ denotes the fundamental solution for Helmholtz problems, in which $r = |P - P_0|$ is the distance between source point P_0 and field point P. $q(P) = \frac{\partial\phi(P)}{\partial n}$, $\phi^I(P_0)$ denotes a prescribed incident wave but it does not exist in radiation problems. Coefficient $c(P_0)$ is described as:

$$c(P_0) = \begin{cases} 1 & P_0 \in E \\ \dfrac{1}{2} & P_0 \in S, \\ 0 & P_0 \in B \end{cases} \tag{5}$$

where E is the exterior region (acoustic medium). S denotes the boundary which is smooth around P_0. B is the interior region (a body or scatterer).

The discretized form of the Eq. (4) can be obtained as the following forms:

$$\sum_{j=1}^{M}\sum_{\alpha=1}^{N^E} H_{ij}^{\alpha}\phi_\alpha = \sum_{j=1}^{M}\sum_{\alpha=1}^{N^E} G_{ij}^{\alpha}q_\alpha + b_i, \quad \text{for node} \quad i = 1, 2, \ldots N \tag{6}$$

here b_i comes from the incident wave for the scattering problems, N denotes the total number of nodes, and

$$\begin{cases} H_{ij}^{\alpha}\phi_\alpha = \left[\displaystyle\int_{S_j}\frac{\partial G(P_i, P)}{\partial n(P)}N_\alpha(P)dS(P) + \sigma(P_i, P_\alpha)c(P_i)\right]\phi_\alpha \\ G_{ij}^{\alpha}q_\alpha = \left[\displaystyle\int_{S_j} G(P_i, P)N_\alpha(P)dS(P)\right]q_\alpha \end{cases} \tag{7}$$

here the S_j denotes the element j, and if the α th node in the element j coincides with the ith node, $\sigma(P_i, P_\alpha) = 1$; else $\sigma(P_i, P_\alpha) = 0$.

3. Multi-frequency calculation technique

In this section, a multi-frequency calculation technique is described. As we all know, the final global coefficient matrices of acoustic problems in BIEM have the wave number dependent character, therefore, the onerous numerical integration is introduced into the computation. To avoid this shortcoming, the wave number should be separated from the fundamental solution $G(P_0, P, k) = \frac{e^{ikr}}{4\pi r}$.

Firstly, we review the method proposed in Li [18]. Li employed the least-squares approximating polynomial for sin(x) and cos(x) on a interval $x \in [0, 5]$ in the following forms:

$$\begin{cases} \sin(x) \approx P_S(x) = a_0x + a_1x^3 + a_2x^5 + a_3x^7 \\ \cos(x \approx P_c(x) = b_0 + b_1x^2 + b_2x^4 + b_3x^6 \end{cases}, \tag{8}$$

Here

$$\begin{cases} a_0 = 0.\ 983652676098689 \\ a_1 = 0.\ 1567147943100234 \\ a_2 = 0.0066827929787795564 \\ a_3 = 0.00009216350206032148 \end{cases} \tag{9}$$

$$\begin{cases} b_0 = 0.\ 9710932877480939 \\ b_1 = 0.\ 4565802435114196 \\ b_2 = 0.0314245577791617 \\ b_3 = 0.0005761784011113857 \end{cases} \tag{10}$$

Thus, the exponential function e^{ikr} can be approximated by Eq. (8). In Li's method, the kL is set as $0 < kL \le 5$, L is the max length of structure. The numerical integration in Li's method is only confined to a frequency-independent part and the obtained boundary element global matrices are frequency independent. The final global coefficient matrices can be simply formed by a summation of the frequency-independent global matrices. However, the approximating for e^{ikr} in Li's method equal to a 7 order polynomial,