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An explicit time integration scheme of numerical manifold method

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ABSTRACT

The traditional numerical manifold method (NMM) has the advantage of simulating a continuum and a discontinuum in a unified framework based on a dual cover system. However, since an implicit time integration algorithm is used, the computational efficiency of the original NMM is very low, especially when more contacts are involved. The present study proposes an explicit version of the NMM. Since a lumped mass matrix is used for the manifold element, the accelerations by the corresponding physical covers can be solved explicitly without forming a global stiffness matrix. The open–close iteration is still applied to ensure computational accuracy. The developed method is first validated by two examples, and a highly fractured rock slope is subsequently simulated. Results show that the computational efficiency of the proposed explicit NMM has been significantly improved without losing the accuracy. The explicit NMM is more suitable for large-scale rock mass stability analysis and it deserves to be further developed for engineering computations in rock engineering.

1. Introduction

Discrete element methods have the advantage of simulating discrete block systems under different loading conditions, which have been widely applied in rock engineering. There are two major representatives in the discrete element method family. In the distinct element method (DEM hereafter) explicit time integration scheme based on finite difference principles is adopted [1]. The DEM has been developed into the commercial 2D code UDEC, 3D code 3DEC and the particle versions PFC2D and PFC3D by Cundall and his colleagues [2–4], which has been enjoying a wide application range in rock engineering. The main benefit of the DEM is that its computational efficiency is high due to its explicit time integration nature. The low computational cost of DEM is mainly because the explicit time integration algorithm does not involve the solution of coupled system equation, so fewer computations are needed per time step [5]. However, it has also been argued that the accuracy of simulated results may be sacrificed in some particular cases. To ensure numerical stability, a DEM simulation requires that the time step must be small enough.

On the other hand, the discontinuous deformation analysis (DDA) derived based on the variational method takes the benefit of the implicit time integration method [6,7]. The formulation of DDA is similar to that of the finite element method (FEM). Due to its implicit time integration, the DDA is unconditionally stable and it is expected to accommodate considerably large time steps. Additional features include the simplex integration method, which is closed-form integration for the element and block stiffness matrix, and the open–close iteration (OCI) contact algorithm. The DDA method has emerged as an attractive approach because its advantage in simulating a discrete system cannot be replaced by continuum-based methods or explicit DEM formulations. Since the initiation of the DDA, many developments and applications have been implemented by Ohinishi et al. [8], Hazator et al. [9], Zhao et al. [10], and others. The major drawback of DDA is that very high computational cost is required, especially when the system contains a huge number of discrete blocks and contacts. The convergence efficiency of the OCI for a complex discrete system is also not clear. This has been a long time challenge for the development of the 3D DDA method. A number of articles on DEM and DDA have been published; detailed mathematical formulations and discussions of the DEM and DDA can be found in the state-of-the-art articles by Jing [11,12] and the book by Jing and Stephansson [13].

The numerical manifold method (NMM) [14,15] involved in this study is an evolution of the DDA, which combines the merits of both FEM and DDA. The NMM inherits all the attractive features of the DDA, such as the implicit time integration scheme, the contact algorithm and the minimum potential energy principle. It adopts a dual cover system, i.e. a mathematical cover overlapping the domain of interest and a physical cover which considers the contained discontinuities and physical boundaries such as material joints, voids, interfaces and aggregates in a unified manner. In the past two decades, many developments have been carried out to improve the performance of the NMM in discontinuous stress problems [16,17], crack propagation problems [18–20], high order NMM theory [21,22], etc. The recent developments of the NMM

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have been reviewed by Ma et al. [16]. It has been recognized that the NMM has great potential to be further developed in simulating massive discontinuities. While possessing the benefits of the FEM and DDA, the NMM suffers from the high computational costs arising from the inherent implicit time iteration scheme and the OCIs for contacts. The implicit time integration algorithm involves the solution of a global system equation, in which the computational cost increases dramatically with increasing of degrees of freedom (DOFs) of the system since the large-scale simultaneous algebraic equations must be solved in each time step. The OCI requires no-tension and no-penetration at all contacts, which additionally increases the computation cost in achieving a convergence state at each time step. When the tolerance of the penetration distance is restricted to a small quantity the time step in the OCI has to be correspondingly reduced, which erases the benefit of using a large time step in an implicit time integration algorithm.

It has been proved that an explicit time integration scheme yields as accurate results as an implicit one if the time step is small enough [5,23]. For a discrete system, the no-tension and no-penetration requirement determines the accuracy of the simulation results. Considering that the computational accuracy is as important as the computational efficiency for engineering problems, a proper balance between the high computational efficiency based on an explicit time integration scheme and the high accuracy based on an appropriate OCI process is highly demanded.

In the present study, a modified version of the NMM based on an explicit time integration algorithm is derived. The original NMM based on displacement method is revised into an explicit formulation of a force method. The governing equations are built up on the dual cover system and the global stiffness matrix used in the traditional NMM is no longer necessary. A diagonal mass matrix is derived for the dual cover system which makes the solution highly efficient at each time step. The OCI is still employed; however, the relative cost is much lower because of the explicit time integration scheme without solving of simultaneous algebraic equations at each step and the smaller penetration incurred due to a smaller time step used. The developed method is validated by two examples: one static problem of a continuous simply supported beam, and one dynamic problem of a single block sliding down on a slope. Results showed that accuracy of the explicit numerical manifold method (ENMM) can be ensured when the time step is small for both the continuous and the contact problems. A highly fractured rock slope is subsequently simulated as well. It is shown that the computational efficiency of the proposed ENMM can be significantly improved, without losing the accuracy, compared with the implicit version of the NMM. The ENMM is more suitable for large-scale rock mass stability analysis and it deserves to be further developed for engineering computations in rock engineering.

2. Fundamental theory of NMM

The traditional NMM is based on the dual cover system, which consists of mathematical covers (MCs), physical covers (PCs) and manifold elements (MEs). The MCs are user-defined small patches, and their union covers the entire problem domain. The PCs are the subdivision of the MCs by the physical features such as the external boundaries and the internal discontinuities, and each PC inherits the partition of unity function from its associated MC. The ME is defined as the common region of several PCs. On each ME, partition of unity functions is used as well to assemble all the local functions associated with the PCs to offer a global approximation.

For a two-dimensional problem a regularly structured mesh is employed in the NMM to form the cover system, which is similar to that in the FEM. As shown in Fig. 1, the structured mesh-based cover system is built on a triangular finite element mesh, in which each node is termed as a ‘star’. The union of six triangles sharing a common ‘star’ forms a hexagonal MC. For the continuous media, each PC coincides with the corresponding MC; thus each MC generates a PC, in which a local function is assigned. Each triangular element e is constructed by the associated three PCs starred at its three nodes. When the discontinuities (i.e. ① and ②) are taken into account in the problem domain, a MC can be subdivided into two and more PCs share the original ‘star’ (i.e. 2 PCs and 4 PCs). If one MC is not or partly cut by the discontinuities, only one PC is constructed. In this case, the refining mesh technique and cutting off the discontinuity tips by the element edges are usually applied to avoid singular matrices occurrence to the utmost extent. For the discontinuities problems, the new generated PCs will be reallocated local functions and updated new indices. Since each MC possesses two DOFs, each element formed by the three overlapping PCs has six DOFs.

Here, a simple example is given to illustrate the constructions of the PCs and MEs on the cover system as plotted in Fig. 2. In the continuum as shown in the left of the figure two interconnected MEs e_i and e_j share two PCs indexed (2) and (3), in which e_i is constructed by the associated three PCs indexed by (1), (2) and (3), and e_j is constructed by the PCs indexed by (2), (4) and (3), respectively. Since the global approximation involves an assembly of a global stiffness matrix, the interaction between e_i and e_j can be

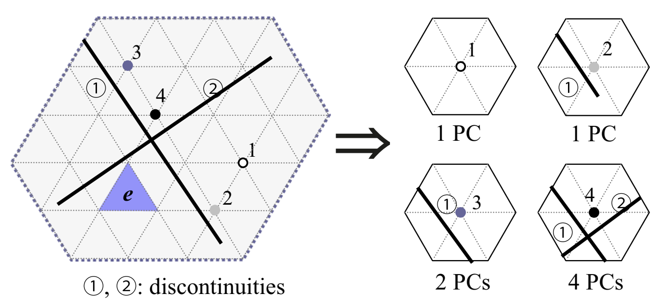

Fig. 1. Structured mesh-based cover system in the NMM.

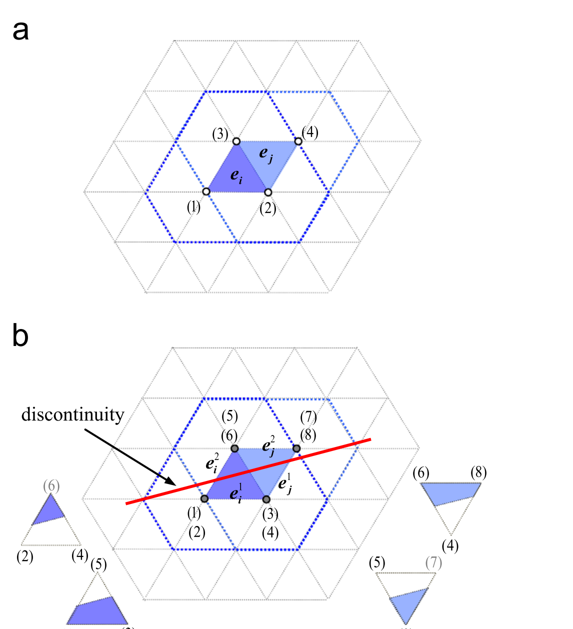

Fig. 2. Construction of manifold elements on the cover system: (a) continuous elements and (b) discontinuous elements.