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Approximation schemes of stresses on elements for the three-dimensional displacement discontinuity method

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ABSTRACT

The displacement discontinuity method is a boundary element method. It uses the analytical expressions for displacements and stresses in an infinite isotropic homogeneous linear elastic body caused by difference (discontinuity) of displacements across small planar crack surfaces. The basic solution of the method is the displacement discontinuities (DDs) across the crack elements. After DDs are obtained, the displacement and stresses at other points in the body can be calculated. It discretises the crack without considering the individual surface of the crack, thus for crack propagation issues, it uses fewer (half) number of elements than normal BEM and therefore less computation time and computer memory requirement. However, it is found that the stresses calculated from the DDs for points on and close to the crack have large errors. Here we present two numerical schemes for approximation of stresses on the crack elements in three-dimensional problems, which are implemented in a code for fracture propagation. The schemes give a reasonably accurate approximation for elements where the crack surface is relatively smooth. It is found that for elements next to sharp kinking or at the corner of a crack, the results from the schemes are not satisfactory. A modification is proposed for these cases.

1. Introduction

Numerical simulations play a very important role in many engineering problems, especially in those where physical experiments are impossible, such as in mining projects. There are many numerical simulation software packages available and they have their advantages and disadvantages. Most common software packages are based on Finite Element Method (FEM), Boundary Element Method (BEM), Finite Difference Method (FDM) or Finite Volume Method (FVM). Recently Discrete (Distinct) Element Method (DEM) has attracted a large amount of attention too. Within each of these methods, there are different branches from different theories, formulations or schemes. Displacement Discontinuity Method (DDM) is a branch of BEM.

Boundary element method employs the analytical expressions for displacement and stress at interior points of a linear elastic body in terms of some variables on the boundary of the body. Displacements and stresses inside a three-dimensional infinite isotropic homogeneous linear elastic body caused by displacement dislocation (or displacement discontinuity—DD) across a small planar crack surface in the body were found analytically by Rongved [1] and this forms the foundation of three-dimensional DDM. The fundamental solution was obtained by using four special harmonic functions. There are other methods for the formulation, such as the boundary integral method [2]. In two-dimensional DDM, displacement and stresses are expressed in terms of displacement discontinuity across line crack segment, see Crouch and Starfield [3] for the theory and a computer code. The basic two-dimensional DDM has been extended to couple hydraulic and thermal effects and to handle multiple material regions, see [4]. In three-dimensional cases, the displacement discontinuity has three components. The displacements and stresses are expressed as weighted integrations of the displacement discontinuity components over the planar crack. If the displacement discontinuity components are uniform over the crack, then the displacements and stresses become linearly dependent on the DDs, see [5–8]. Various aspects related to three-dimensional DDM have been studied previously by several researchers [9–12], including higher order elements [9,10] specially for crack front edge. DDM has also been used in poro-elasticity, thermo-elasticity and electro-elasticity theories [13–15].

If displacements or stresses at some positions are known, then the displacement discontinuity at the planar crack that produces the known displacements or stresses can be solved inversely and the displacements and stresses at other positions can be calculated from the analytical expressions. If the crack is curved, non-planar,

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it is approximated as an assembly of a finite number of planar crack elements. The displacements and stresses at a point are then taken as the sum of the effects from all the planar elements. This superposition is possible since the material is assumed to be linear elastic. If the displacements and/or stresses at sufficient number of positions are specified, then the displacement discontinuities (DDs) at the planar elements that cause the specified displacements and/or stresses can be found by solving a system of equations. The definition of a crack can be extended to include an "imaginary" crack, formed on the boundary surface of a finite body or cavity surface of an infinite body by imagining the region outside the finite body or the cavity being filled with the same material as the body considered. We are developing a code, FRACOD^{3D}, which uses the three dimensional DDM with triangular elements.

It was found that the stresses calculated from the DDs for points on or close to the crack elements have large errors after the DDs are determined from the basic linear equations. Two dimensional DDM also has this problem, see [3,4,16]. Some other BEMs, such as those using boundary integral equations [17] do not have such a problem for stresses on the boundary elements. In this paper, we present some schemes to calculate the stresses on the crack element approximately. In Section 2, we show which stress components need the approximation. Section 3 illustrates a scheme (scheme 1) to calculate the stress components from three neighbouring elements, while calculation from two neighbouring elements (scheme 2) is presented in Section 4. It is noted that these schemes can be used for triangular and quadrilateral elements. Section 5 discusses implementation of these schemes in FRACOD^{3D} and some numerical examples are given in Section 6, which use the combination of scheme 1 and 2.

2. Stresses on boundary elements from DDM

With DDM, linear equations for the displacement discontinuity components (DDs) are first set up according to the given conditions on the collocating boundary elements. The coefficients in the matrix of the equations represent the influence of DDs of elements on the collocating elements. After DDs are solved from the equations, the displacements and stresses at other required points can be calculated. However, if the points are too close to or on the boundary elements, then the stresses calculated in this way have large errors.

Although the discretisation for DDM is on crack as one unity, not on its two surfaces, the elements are considered to have two faces when displacement and stresses are calculated. While the displacements are discontinuous across the crack surfaces, the stress components on the element faces are continuous across the crack surfaces. It is commonly known that stress state at a point in an isotropic body is given by six stress components. Fig. 1 illustrates the generic three-dimensional stress state at a point on one face of an element on the crack. For three-dimensional DDM, the stress components on the element face, $\sigma_{xz}, \sigma_{yz}, \sigma_{zz}$ in Fig. 1 are known as the specified boundary values for traction boundary value problems or can be calculated through the formation of the influence coefficients for the basic equations. These components are continuous across a crack with the DDM formulation. The stress components on the planes orthogonal to the element face ($\sigma_{xx}, \sigma_{yy}, \sigma_{xy}$ in Fig. 1) are not known and some numerical schemes for their calculation are needed to avoid the large errors. Two approximate numerical schemes will be outlined in the following. It is noted that these stress components are not continuous across the crack with the DDM formulation.

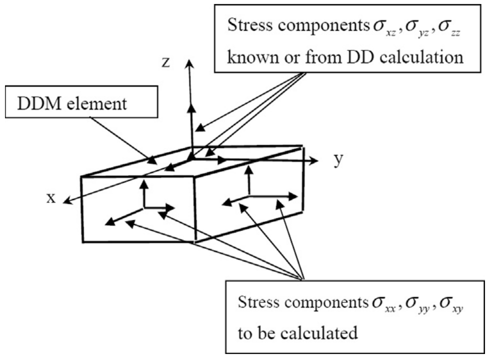

Fig. 1. Stress components known and to be calculated at a point on a DD element face.

In the local coordinate system on the element, the three stress components on the element face are $\sigma_{zx}, \sigma_{yz}, \sigma_{zz}$. The other three stress components at the elemental point are $\sigma_{xx}, \sigma_{yy}, \sigma_{xy}$. At points on the two different surfaces of a crack, $\sigma_{xx}, \sigma_{yy}, \sigma_{xy}$ are not continuous and they are denoted as $\sigma^{\pm}_{xx}, \sigma^{\pm}_{yy}, \sigma^{\pm}_{xy}$. From the generalised Hooke's law, it can be seen that these stress components are given as

$$\sigma_{xx} = \frac{E}{1-\nu^2}(e_{xx} + \nu e_{yy}) + \frac{\nu}{1-\nu}\sigma_{zz},$$
$$\sigma_{yy} = \frac{E}{1-\nu^2}(e_{yy} + \nu e_{xx}) + \frac{\nu}{1-\nu}\sigma_{zz},$$
$$\sigma_{xy} = 2Ge_{xy}, \tag{1}$$

where E, ν, G are Young's modulus, Poisson ratio and shear modulus, respectively, of the material. The stress component σ_{zz} is equal to the normal traction on the element face and is known as the specified traction boundary value or can be calculated through the influence coefficients. For computation of these stress components,

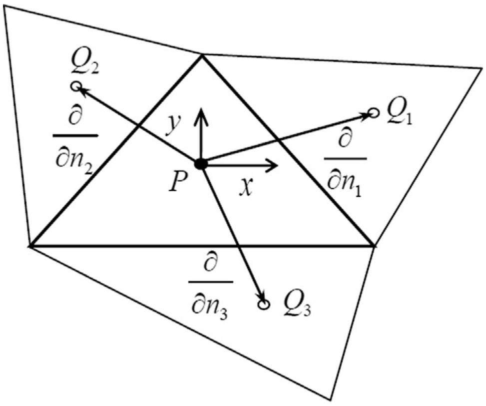

Fig. 2. Calculating stresses for element with three neighbours.

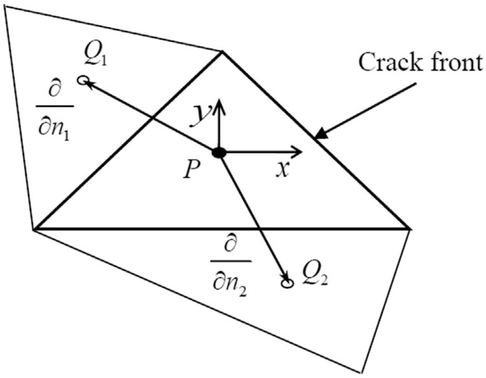

Fig. 3. Calculating stresses for element with two neighbours.