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A three-dimensional crack growth simulator with displacement discontinuity method

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ABSTRACT

This paper first outlines the theory of a well established three dimensional boundary element method: displacement discontinuity method (DDM) and proposes to use a crack growth criterion based on maximum normal or shear stress for a three dimensional crack growth simulator, FRACOD^{3D}. Triangular elements are used in the simulator code. A numerical scheme is used to overcome a difficulty associated with the evaluation of the basic solution for DDM in some special situations and another numerical scheme is used to calculate the stresses on the boundary elements where the stresses obtained from the normal DDM scheme have large errors. The crack growth is implemented incrementally in that new front elements are introduced at the crack front; thus no need to re-mesh the old part of the cracks. The effects of neighbouring front elements are taken into account in implementation of the crack growth to overcome severer twisting of the new front elements generated from the growth. The numerical results from FRACOD^{3D} of two simple examples agree very well with analytical solutions, and propagation configuration of a circular disc crack in an infinite body under shear is close to that observed in an experiment in literature under similar loading condition.

1. Introduction

Numerical simulation becomes essential for engineering problems for which physical experiments are very difficult or even impossible, such as crack propagation around mining sites. There are several classes of numerical simulation methods, including Finite Difference Method (FDM), Finite Element Method (FEM), Boundary Element Method (BEM), Finite Volume Method (FVM) and Discrete Element Method (DEM). FEM may be the most popular method, but others have their advantages. One advantage of boundary element methods over finite element methods is that BEM uses formulations of one dimension less than others: one-dimensional formulation for two-dimensional problems and two-dimensional formulation for three-dimensional problems, because it only deals with the boundary of the problems. So generally the number of final equations in BEM is smaller than that of FEM for the same problem. Displacement discontinuity method (DDM), a BEM, for crack problems has at least one advantage over other BEMs in that the DDM treats the crack as one surface while other BEMs have to consider discretisation of both surfaces of the crack. Therefore the DDM further reduces the number of the final equations. But one should note that the final system of equations from BEM has a dense coefficient matrix, or even a full matrix in most cases, different from the sparse matrix from FEM and thus the solution schemes for sparse system of equations cannot be used.

DDM is based on the fundamental solution for displacements and stresses in infinite elastic bodies caused by displacement dislocation (displacement discontinuity – DD) on surfaces of a finite planar crack inside the body. The fundamental solution is valid for isotropic, homogeneous and linear elastic material. If the crack is curved and non-planar, then it is approximated as an assembly of a finite number of planar crack elements. The displacements and stresses at a position inside the body are then taken as the sum of the effects from all the planar elements. This superposition is possible since the material is assumed to be linear elastic. For problems of finite body or cavity in infinite body, the surface of the finite body or surface of the cavity can be treated as imaginary closed curved crack in an infinite body, with the region outside the finite body or the cavity being filled with the same material, allowing for the fundamental solution to be employed.

Generally, in practice the displacement discontinuities at the elements are not known, so the fundamental solution cannot be used directly. If the displacements and/or stresses at a sufficient number of positions are specified as the boundary condition, then the displacement discontinuities at the planar elements that cause

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the specified displacements and/or stresses can be found by solving a system of equations. The displacements and stresses at other positions in the body can then be calculated from the obtained displacement discontinuities on the elements. This is the basic principle of DDM.

In two dimensional cases, Crouch and Starfield [1] outlined the theory and implemented the method in computer code, which is supplied in the book. Crouch and Starfield [1] also presented the governing equations for three dimensional situations. Shen [2] used the DDM to simulate crack growth with a newly proposed criterion for rock masses. The basic two dimensional DDM has been extended to couple hydraulic and thermal effects and to handle multiple material regions, see [3]. Various ways have been used to derive the governing equations of DDM. For example, Hong and Chen [4] showed that DDM is a special case of dual boundary element method. In three dimensional cases, Rongved [5] obtained the fundamental solution by using four special harmonic functions. There are other ways for the formulation [6]. Many papers on three dimensional DDM exist, see [7–12] for different aspects of the method. DDM has also been employed in three dimensional poroelastic and therm-elastic problem modelling [13,14].

The core of DDM is integral functions, integrals over the planar crack region of products of Green function and the displacement discontinuity components [5]. If the displacement discontinuity components are uniform over the crack region, then the integral functions depend on one single integral. In this case, to calculate displacements and stresses in the body, the derivatives of the integral function are needed. The analytical expression of the integral function has been obtained for a triangular element (see [15,7]) and its partial derivatives can be computed analytically, see [8] for those derivatives needed in DDM. However, if the perpendicular projection of the considered point on the integration plane is on a line along which a side of the triangular integration element lies, then the integral function and its derivatives become infinite. This makes the evaluation of these expressions difficult or impossible. This was reported by Kuriyama and Mizuta [8], but they did not provide an alternative scheme. Instead they tried to arrange elements in their examples so that this issue did not occur. It is noted that the whole analysis breaks down if this occurs on just any one element. This cannot be avoided for some of analysed domains, such as rectangular block and cylinder surfaces. In addition, when propagation of cracks is considered, it is very hard to avoid this occurring on elements on newly extended crack surface. In this paper, we propose to use a numerical integration scheme for such cases, or even for regular points. We also discuss various methods for computing the derivatives. In addition, we employ a numerical scheme to calculate stresses on the elements, where the normal DDM procedure yields large errors.

Cracks exist in natural rock medium so a computational code used for simulating rock medium needs to be able to simulate crack growth in cases where crack front stresses exceed the rock strength. Three-dimensional crack growth has a very complicated mechanism. Various theories exist to describe crack growth, such as maximum principal stress criterion and minimum strain energy density factor criterion [16,17]. With these theories there are three basic modes in which a crack can grow: open mode (Mode I), shearing mode (Mode II) and tearing mode (Mode III). In practice, a crack could propagate in a way which combines these basic modes. A modified criterion based on maximum stress is proposed here to determine whether or not a crack will grow, and if it grows, in which direction it will go.

DDM is based on the displacement discontinuity of cracks, so it should be suitable for crack growth simulation. Although there is a large number of literatures on crack growth simulation in two-dimension using DDM, there are not many papers in three-dimensional cases, as far as the authors are aware, possibly due to the difficulty of eliminating or reducing severe twisting of front elements generated from the growing crack surface after a few steps of propagation with general loading conditions [18]. This difficulty exists also in other numerical methods [19]. The difficulty is even worse when multiple cracks grow and their growths meet other cracks. This will be addressed separately in a future work. A paper published in Journal of Donghua University (2010) by Wang and Huang was brought into our attention recently. Wang and Huang gave some implementation details of 3D crack growth, together with basis of DDM and crack propagation criterion.

In this paper we describe the theory and implementation of a simulation code FRACOD^{3D}, based on three dimensional DDM, and the proposed criterion for crack growth. We attempt to make the code robust for general situations. The crack growth is implemented incrementally in that new front elements are introduced at the whole crack front step by step. In this way, mesh of the old part of the cracks is kept at each step of growth. The effects of neighbouring front elements are taken into account in implementation of the crack growth to overcome severer twisting of the new front elements generated from the growth. It is found in the present paper that this scheme works well. The DDM part of the code is verified with two examples for which analytical solutions exist. These include (1) circular planar crack under tension in the direction normal to the crack plane and (2) spherical cavity under uniform inner pressure on the cavity surface. Capability of the code for simulation of crack growth is shown with one example of circular planar crack under shear.

The paper is arranged as follows. Section 2 outlines the fundamental solution due to displacement discontinuity on a crack and Section 3 gives the details of how to use the fundamental solution to solve practical problems. Various ways of evaluating the coefficients in DDM are outlined in Section 4. Section 5 formulates the criterion for crack growth. Implementations of DDM and the crack growth are described in Sections 6 and 7, respectively, before two verification examples and one crack growth example are shown in Section 8. Finally, some conclusions are given in Section 9.

2. Three-dimensional elasticity with displacement discontinuity

The fundamental solution of DDM is for displacements and stresses at any point in an infinite body caused by known displacement discontinuity on a planar crack in the infinite body. To express the solution mathematically, a local coordinate system, *oxyz*, is set up such that the planar crack is located on the *oxy* plane, as shown in Fig. 1. Let the displacement components on the lower surface of the crack, which has outward normal in positive z direction, be u_x^+, u_y^+, u_z^+, and let the displacement components

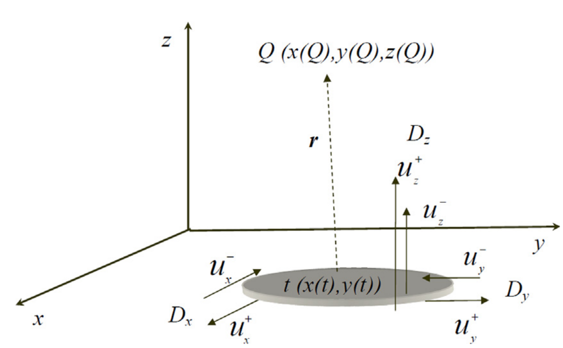

Fig. 1. Coordinate system and components of displacement discontinuity.