Engineering Analysis with Boundary Elements 60 (2015) 10–17

Contents lists available at ScienceDirect

Engineering Analysis with Boundary Elements

journal homepage: www.elsevier.com/locate/enganabound

Free vibration analysis of two-dimensional functionally graded coated and undercoated substrate structures

Y. Yang*, K.P. Kou, C.C. Lam, V.P. Iu

Department of Civil and Environmental Engineering, Faculty of Science and Technology, University of Macau, Macau, China

ARTICLE INFO

Article history:
Received 24 September 2014
Received in revised form
25 March 2015
Accepted 17 April 2015
Available online 27 May 2015

Keywords:
Free vibration
FG coated and undercoated substrate structures
Boundary–domain integral equations
Meshfree method
Multi-region boundary element method

ABSTRACT

In this paper, the free vibration behaviors of the functionally graded (FG) coated and undercoated substrate structures are studied by a meshfree boundary–domain integral equation method. Based on the two-dimensional elasticity theory, the boundary–domain integral equations for each single layer of these coating–substrate structures are derived initially by using elastostatic fundamental solutions. Employ the radial integration method to transform the domain integrals into boundary integrals and achieve a meshfree scheme. By applying the multi-region boundary element method, obtain the generalized eigenvalue system of the whole structure, which involves system matrices with boundary integrals only and the complete solutions for natural frequency and vibration modes are rigidly resolved. A comparative study of FG versus homogeneous coating is conducted. The influences of material composition, material gradient, coating thickness ratio, substrate structure aspect ratio and the boundary conditions on the natural frequencies of the FG coated and undercoated substrate structure are evaluated and discussed.

1. Introduction

In many applications, especially in the space industry, energy industry and electronic industry, structures or part of structures are exposed to high temperature or high temperature gradients. Conventional metallic materials, such as carbon steels or stainless steels cannot resist such high temperature. In order to improve the resistance of metallic structures against extreme temperature conditions, without suppressing their strength and toughness, a thin layer of appropriate ceramic is generally used to cover the surface of the structures. For those structures which are subjected to constantly rolling, sliding contacts or abrasive wear, additional hardening process should be carried out within the outer surface of the materials. These two techniques are all forming the coating–substrate system, where a functional material is coated on the substrate material to increase the durability and reliability of the structures. However, due to the discontinuous of the material properties of these two or more materials, severe residual and working stresses discontinuity at the material interfaces usually cause damage to the coating, or failure due to delamination. As a remedy to the aforementioned disadvantages in coating–substrate system, a concept of functionally graded (FG) coating is proposed, where a smooth spatial gradation of the material properties is introduced from coating to substrate in order to eliminate the effect of the sudden change of the material properties, such that stress and strain discontinuous can be mitigated in the coating–substrate system.

Due to the superior properties, a world-wide requirements of the application of the FG coating–substrate system triggered a series of research activities. The incorporation of functionally graded materials (FGMs) into coating design can help eliminate the mismatch of mechanical and thermo-mechanical properties between the metal plates and coating layers. Thus a number of studies existed in the literature for analyzing of the mechanical and thermo-mechanical behavior of homogeneous plates coated by a FG layer. A FG coated elastic solid under thermomechanical loading was carried out by Shodja and Ghahremaninejad [1]. Three-dimensional elastic deformation of a functionally graded coating/substrate system was investigated by Kashtalyan and Menshykova [2]. Chung and Chen [3] analyzed the bending behavior of the thin plates coated by FG layer. Several researches have addressed contact response of FG coatings. Saizonou et al. [4] studied the subsurface stress distribution of an FG-coated elastic solid under normal and sliding contact loading by the boundary element method (BEM). Contact mechanics of the FG coated solids was analyzed by Guler and Erdogan [5]. It should be noted that in all above studies the properties the FGM were all assumed to vary exponentially through the thickness.

Theoretical modeling of FG coatings has been focused predominantly on prediction of their fracture behaviors. Chen and

* Corresponding author.
E-mail address: yangyang.liju@hotmail.com (Y. Yang).

http://dx.doi.org/10.1016/j.enganabound.2015.04.009
0955-7997/

Erdogan [6] studied the interface cracks for a FG coating medium. A crack in the FG coating surface and its expansion into the substrate along the direction perpendicular to the interface between the coating and the substrate was presented by Chi and Chuang [7]. Pindera et al. [8,9] examined fracture mechanisms in thermal barrier coatings with FG bond coats under uniform cyclic thermal loading. However, the dynamic analyses of the FG coatings are very rare in the literature. Liew et al. [10] investigated linear and non-linear vibrations of a coating–FGM–substrate cylindrical panel subjected to a temperature gradient, which were based on the first order shear deformation theory and von-Karman geometric nonlinearity. Hosseini-Hashemi et al. [11] presented the exact closed-form solutions for both in-plane and out-of-plane free vibration of the simply supported rectangular plates coated by a FG layer, based on three-dimensional elasticity theory.

In this paper, attention is focused on investigating the free vibration behaviors of two FG coating–substrate structures. The first one involves a two-layer structure, namely an FG layer coated on a homogeneous substrate which is simply called the FG coated substrate structure, the other involves a three-layer structures in which a FGM is employed for the inter-medium layer and different homogeneous materials are in the top and bottom layers, this is called an FG undercoated substrate structure [3]. For each single layer of these structures, the boundary–domain integral equation formulations are derived initially by using the elastostatic fundamental solutions which is based on the two-dimensional elasticity theory. A meshfree scheme is achieved to apply the radial integration method to transform the domain integrals arising from the material inhomogeneous and the inertial effects to the boundary integrals. Finally, an eigenvalue system involving system matrices with boundary integrals only is obtained through assembling all the sub-layer integral equations together by employing the multi-region BEM. By the harmonious combination of this meshfree boundary–domain integral equation method and the multi-region BEM, a comparative study of FG coating versus homogeneous coating is conducted. Numerical example results are presented to demonstrate the influences of FG coating thickness ratio, substrate structure aspect ratio, as well as boundary condition on the vibration characteristics of the FG coated and the FG uncoated substrate structures.

2. Material properties of the coating–substrate structures

Three considered coating–substrate structures, namely, the homogeneous coated, FG coated, as well as the FG undercoated structures are schematic depicted in Fig. 1. Assume the layers of these coating–substrate structures are perfected bonded to each other. The total length and height of these coating–substrate structures are denoted by L and h_t. h_i represents the thickness of each layer. The coating and the substrate of the homogeneous coated structure as well as the top and the bottom layers of the FG undercoated structure are composed by pure ceramic and pure steel, respectively, their material parameters are described in Table 1. For the FG layer existing in FG coated and FG undercoated substrate structures, assuming the top is ceramic rich and the bottom is steel rich, Young's modulus and the mass density are varying continuously in the transverse direction according to an exponential function described in Eqs. (1) and (2), while the Poisson ratio is constant.

$$E(x_2) = E_b e^{\beta x_2} \quad \text{where} \quad \beta = \frac{1}{h_f} \ln\left(\frac{E_t}{E_b}\right) \tag{1}$$

$$\rho(x_2) = \rho_b e^{\gamma x_2} \quad \text{where} \quad \gamma = \frac{1}{h_f} \ln\left(\frac{\rho_t}{\rho_b}\right) \tag{2}$$

where E_t, ρ_t are Young's modulus and mass density for the top face constituent of the FG layer, and E_b, ρ_b are for the bottom face constituent. FGM gradation parameters are represented by β and γ for Young's modulus and mass density respectively. x_2 denotes the Cartesian coordinates variable in the transverse direction and h_f is the thickness of the FG layer. The through thickness variation of Young's modulus for the three considered coating–substrate structures is shown in Fig. 2.

3. Problem formulation

The fulfillment of the free vibration analyses of the FG coated and FG undercoated substrate structures as well as the homogeneous coated substrate structures are by the harmonious combination of the developed meshfree boundary–domain integral equation method and the multi-region BEM.

3.1. The meshfree boundary–domain integral equation method

For each single layer of the coating–substrate substrate structures, the governing differential equations of the steady-state elastodynamics without damping is expressed in terms of the frequency ω as

$$\sigma_{ij,j}(\boldsymbol{x}) + \omega^2 \rho u_i(\boldsymbol{x}) = 0 \tag{3}$$

which is based on the two-dimensional elasticity theory and the stress tensor σ_{ij}, mass density ρ, displacement u_i are quantities for each layer. A comma after a quantity represents spatial derivatives and repeated indexes denote summation.

The elasticity tensor c_{ijkl} is described in the form of

$$c_{ijkl}(\boldsymbol{x}) = \mu(\boldsymbol{x}) c^0_{ijkl} \tag{4a}$$

Table 1
Material properties of the homogeneous ceramic and steel.

Material	E(GPa)	ρ(Kg/m^3)	ν
Aluminum (Al)	70	2707	0.3
Steel (S)	210	7806	0.3

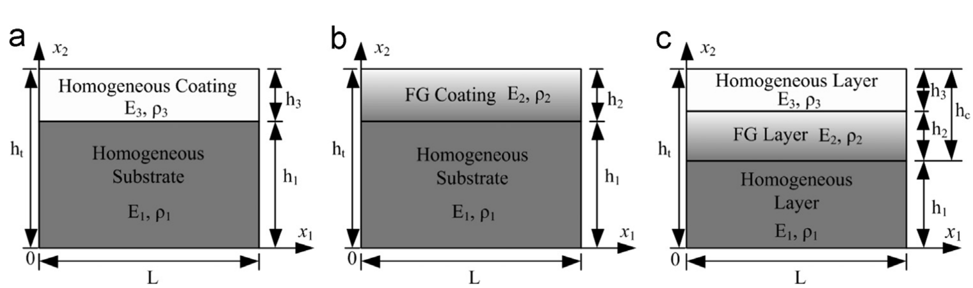

Fig. 1. Coordinates and geometry of the coating–substrate structures. (a) Homogeneous coated substrate structure; (b) FG coated substrate structure; (c) FG undercoated substrate structure.