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An isogeometric enriched quasi-convex meshfree formulation with application to material interface modeling

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ABSTRACT

An isogeometric enriched quasi-convex meshfree method is presented with particular application to the material interface modeling. The current quasi-convexity of the meshfree approximation is achieved by introducing the mixed reproducing points of isogeometric B-spline basis functions into the meshfree consistency conditions. The resulting new meshfree shape functions have a similar form as the standard reproducing kernel meshfree shape functions, while the negative portions of the shape functions are significantly reduced. It is shown that this quasi-convex meshfree scheme yields better accuracy compared with the conventional meshfree method. Furthermore, in order to accurately model the material interface where the strain jump needs to be properly treated, a coupled isogeometric–meshfree approximation with a unified format of reproducing conditions is devised. The problem geometry and strain jump for the material interface are described by the isogeometric basis functions with repeated knots in the interface normal direction, while the rest regions are discretized by the isogeometric enriched quasi-convex meshfree approximation. This approach encompasses the geometry exactness of isogeometric analysis as well as the model refinement robustness of meshfree formulation. The effectiveness of the proposed method is thoroughly demonstrated by several typical numerical examples.

1. Introduction

Meshfree methods [1,2] and isogeometric analysis (IGA) [3,4] are two classes of numerical methods that have attracted significant attention and experienced rapid developments with successfully applications [5–26] recently. The construction of higher order smoothing approximations is trivial in both types of methods, which is not an easy task for the conventional finite element method [27]. Meanwhile, the node based meshfree approximation enables a very flexible and robust local model refinement [5–8]. The isogeometric analysis gracefully integrates computer aided geometry design and finite element analysis, and thus offers model refinement independent geometry exact computations [3,17]. Nonetheless, special treatments like T-splines are necessary for the isogeometric local model refinement due to the tensor product nature of multidimensional basis functions [28]. In order to employ the easy local refinement and geometry exactness advantages of the meshfree methods as well as isogeometric analysis, a consistently coupled isogeometric–meshfree method [29] has been developed, where the consistency conditions of B-spline basis functions were presented along with theoretical and computational verification. We just found that these consistency or reproducing conditions can also be obtained by employing the Marsden's identity for splines [30]. It turns out that these consistency conditions are essential to ensure the optimal convergence of the coupled isogeometric–meshfree method [29,31]. However, in this coupled approach, the meshfree part does not share the convex approximation property with the isogeometric basis functions.

Convex approximation implies non-negative basis or shape functions which give a variation diminishing approximation and non-negative mass matrices for dynamic analysis [32]. The B-spline or NURBS basis functions used in isogeometric analysis are naturally convex [3]. On the other hand, the frequently used moving least square or reproducing kernel meshfree approximants are non-convex [1,2]. Consequently, the construction of convex meshfree approximations is a current focus of meshfree methods [32–39]. The maximum entropy meshfree approximation is a typical convex meshfree approximation, where the shape functions are obtained by minimizing the entropy function under the linear completeness constraints [32–34]. Iterative process is usually required for the shape

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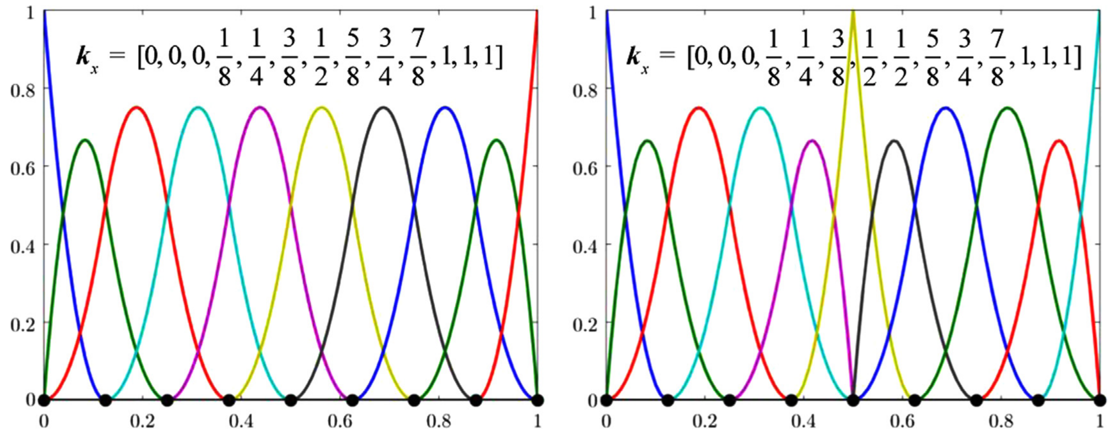

Fig. 1. 1D quadratic C^1 and C^0 B-spline basis functions.

function computation. Another convex meshfree approximation meeting linear completeness was realized by the generalized meshfree approximation scheme [35,36]. Although several attempts were made to extend the max-entropy meshfree interpolant to quadratic order completeness or consistency conditions [38], the construction of arbitrary order max-entropy convex meshfree shape functions is still an open issue. Emanating from a different path, a quasi-convex meshfree method [39] was developed through a compromise between the arbitrary order general formulation and the strict convex requirement. This method relaxes the consistency conditions in a unified manner and admits an easy and straightforward construction of arbitrary order meshfree approximations that exhibit a quasi-convex behavior, i.e., the shape functions for the interior nodes are close to be non-negative and meanwhile the negative values of the shape functions associated with the near boundary nodes are reduced notably [39].

In this work, an alternative approach is proposed to construct quasi-convex meshfree approximation, i.e., the isogeometric enriched quasi-convex reproducing kernel approximation. It is noted that B-spline basis functions show perfect convex property and thus in this study the mixed reproducing points of B-spline basis functions are directly built into the reproducing kernel consistency conditions. It is shown that this formulation also leads to quasi-convex reproducing kernel meshfree shape functions without introducing the nodal gap functions [39]. The accuracy of this new quasi-convex meshfree approximation is demonstrated through several examples. Although the smoothing meshfree approximation is an obvious advantage in many situations, it may become a disadvantage when dealing with the material interfaces arising from the analysis of heterogeneous materials. The higher order meshfree continuity has to be modified via different techniques to accommodate the strain jump across a material interface [40–44]. Clearly exact description of the material interface has an important effect as well. In order to have a geometry exact formulation, the isogeometric approach is adopted to describe the problem geometry as well as the material interface, the rest problem domain is discretized using the proposed isogeometric enriched quasi-convex reproducing kernel meshfree approximation. Thus the strain or gradient jump across the material interface can be readily captured by setting repeated knots in the interface normal direction. The coupling of various regions is achieved by modifying the meshfree approximants through a unified formulation of consistency conditions. The effectiveness of the proposed material interface modeling method is demonstrated by a series of examples.

The layout of this paper is as follows. The basics of isogeometric and meshfree approximations are described in Section 2. Section 3 elucidates an isogeometric enriched quasi-convex meshfree approximation

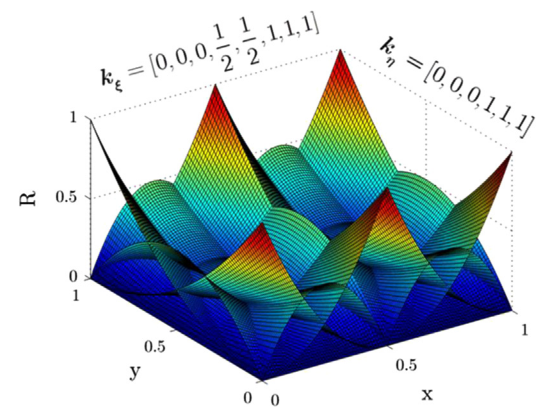

Fig. 2. 2D quadratic C^0 B-spline basis functions.

that directly embeds the mixed reproducing points of B-spline basis functions into the reproducing kernel meshfree formulation to minimize the negative parts of the conventional meshfree shape functions. The modeling of material interface with the isogeometric enriched quasi-convex meshfree formulation is detailed in Section 4. Several examples are presented in Section 5 to verify the proposed algorithm. Finally conclusions are drawn in Section 6.

2. Basics of isogeometric and meshfree approximations

2.1. Isogeometric basis functions

The basis or shape functions commonly used in isogeometric analysis are the B-spline basis functions and their rational generalization, the non-uniform rational B-spline (NURBS) basis functions. The basis functions for multi-dimensional isogeometric analysis are usually formulated through the tensor product operation on their one dimensional counterparts in different directions.

A group of B-spline basis functions with an order of p can be conveniently defined with the aid of the knot vector $\boldsymbol{k}_\xi$ that is formed by a set of non-decreasing numbers in the one dimensional parametric domain $\xi \in [0,1]$:

$$\boldsymbol{k}_\xi = \{\xi_1 = 0, \ldots, \xi_i, \ldots, \xi_{nk} = 1\}^T \tag{1}$$

where nk denotes the number of knots. When the first and last knots are repeated $(p+1)$ times, a knot vector is said to be an open knot vector and in this case $nk = nb + p + 1$, where nb is the number of B-spline basis functions. A pth order B-spline basis