Engineering Analysis with Boundary Elements 60 (2015) 115–122

Contents lists available at ScienceDirect

Engineering Analysis with Boundary Elements

journal homepage: www.elsevier.com/locate/enganabound

Analysis of the temperature field in anisotropic coating-structures by the boundary element method

Changzheng Cheng *, Zhilin Han, Huanlin Zhou, Zhongrong Niu

School of Civil and Hydraulic Engineering, Hefei University of Technology, Hefei 230009, China

ARTICLE INFO

Article history:
Received 31 October 2014
Received in revised form
13 January 2015
Accepted 22 January 2015
Available online 16 February 2015

Keywords:
Anisotropic
Coating-structures
Temperature field
Boundary element method
Nearly singular integral

ABSTRACT

The distance r between the source point and the field point is very short when the boundary element method is used to calculate the boundary quantities in the coating domain, which is very thin with respect to the substrate. The nearly singular integrals will occur during the process of numerical calculation of the boundary integral equations when the distance r is approaching to zero. The calculation difficulty of nearly singular integrals has seriously hindered the application of the boundary element method to the analysis of the physical quantities in the coating-structures. Herein, the analytical formulations for the nearly singular integrals in potential boundary integral equations developed by the authors before are generalized to the multi-domain system. This multi-domain boundary element method, in which the nearly singular integrals have been cracked, is introduced to analyze the temperature field in anisotropic coating-structures. The numerical examples demonstrate that the present method can model the temperature field in the coating-structure with much thinner coating in contrast with the conventional boundary element method. For the cases there are no analytical solutions available, the solutions from the finite element method are given out as the referenced ones. The temperature fields obtained by the present method can approach to the finite element solutions perfectly when the coating is very thin. The present method is versatile for the temperature analysis of the isotropic, orthotropic and anisotropic coating-structures.

1. Introduction

The rapid development of technologies for synthesizing a new generation of anisotropic materials with thermo-physical and mechanical characteristics has opened up broad possibilities for developing the protective coatings. The level of nano-technologies already achieved enables the human being to hope for the creation of nano-coatings to improve machining performance due to better temperature and wear resistant properties than their substrate counterparts [1]. The analysis of the thermal and mechanical properties for coating-structures creates a need to develop new methods or update existed methods for investigating the temperature field in the anisotropic coating-structures.

The finite element method (FEM) is a successful tool for the analysis of thermal and mechanical property for many industrial structures. Nevertheless, the number of elements in the FEM increases dramatically for modeling the coating-structures due to the limitation of the aspect ratio, and the procedure therefore requires much CPU time as the coating thickness decreases. The boundary element method (BEM) is also a powerful and efficient computational method for the analysis of the engineering structures. The main advantage of the BEM is the reduction of the dimension of the boundary value problem. However, it is well known that the BEM is based on the fundamental solution of partial differential equations, which include the distance r between the source point and the field point. The nearly singular integrals will arise when the BEM is applied to analyze the coating-structures, because the source point very closes to the field point in the coating domain. The efficiency of the BEM to analyze the coating-structures depends on the computational accuracy of the nearly singular integrals [2–7].

Some analytical and semi-analytical formulae were proposed for the evaluation of the nearly singular integrals in the boundary integral equations. Most of them are based on the distance transformation technique [8–16]. There are some other new methods proposed by the researchers to deal with the calculation difficulty of the nearly singular integrals. The calculation of 2-D nearly singular integrals based on the complex space C was proposed by Dehghan and Hosseinzadeh [17]. The sinh-sigmoidal method was proposed for the numerical evaluation of nearly singular integrals on triangular boundary elements by Johnston et al. [18]. The iterated sinh technique was extended to evaluate the 2-D nearly singular integrals by Johnston et al. [19]. A new indirect regularized boundary element formulation excluding the Cauchy Principal

* Corresponding author.

http://dx.doi.org/10.1016/j.enganabound.2015.01.010
0955-7997/

Value and Hadamard-Finite-Part integrals was proposed for the numerical computation of the nearly singular integrals in 3-D BEM by Zhang et al. [20,21]. A natural stress boundary integral equation by introducing the natural variables was proposed to analyze the stress field of interior points near the boundary by Cheng et al. [22].

As to the analysis of the temperature field by the boundary element method, Marczak and Denda [23] presented two new methods to derive the fundamental solutions for 3-D heat transfer problems in the general anisotropic media. The singularity analysis method coupled with the boundary element technique was proposed for the accurate analysis of 2-D singular heat flux field near the notch tip by Cheng et al. [24]. A new completely analytical integral algorithm was proposed and applied to the evaluation of nearly singular integrals in the boundary element method for 2-D orthotropic and anisotropic potential problems by Zhou et al. [25,26]. A new robust boundary element method was proposed for solving general anisotropic potential problems with varying coefficients based on a source point isolation technique by Gao [27].

About the analysis of the physical quantities in the coating-structures, a general boundary element technique was developed for determining the temperature fields in thin coatings based on a single-layer approximation technique by Du et al. [28]. An advanced boundary element method was developed for analyzing thin layered structures by Luo et al. [29] and Chen and Liu [30]. A general strategy based on a nonlinear transformation technique was introduced and applied to evaluate the nearly singular integrals for analyzing the thermal behavior in thin-coated cutting tools by Zhang et al. [31].

Most of the research work was focused on the temperature field analysis in the isotropic material or orthotropic material. To the author's best knowledge, there is no work that focuses on the analysis of the temperature field in the anisotropic coating-structures. The analytical integral formulae for the nearly singular integrals proposed by the authors before [26] are introduced to the multi-domain boundary element method to analyze the temperature field in the coating-structures. The present method is validated for analyzing the temperature distribution in the coating-structures with much thinner coating, by contrast with the conventional boundary element method. The present method is a general method, which can be used to determine the temperature field in isotropic, orthotropic and anisotropic coating-structures.

2. Boundary element method for anisotropic coating-structures

For fully anisotropic media, the governing equation for the temperature field T can be written as

$$k_{11}\frac{\partial^2 T}{\partial x_1^2}+2k_{12}\frac{\partial^2 T}{\partial x_1 \partial x_2}+k_{22}\frac{\partial^2 T}{\partial x_2^2}=0 \tag{1}$$

for two-dimensional problems, where the $k_{ij}\,(i,j=1,2)$ terms define the heat conduction coefficients. By using the weighted residual method on Eq. (1), the temperature boundary integral equation is given as

$$C(y)T(y)+\int_\Gamma q^*(x,y)T(x)d\Gamma(x)=\int_\Gamma T^*(x,y)q(x)d\Gamma(x) \tag{2}$$

where $C(y)$ is the coefficient determined by the local geometry of source point y, $T(x)$ and $q(x)$ are the temperature and normal flux of the field point x on the boundary Γ, respectively. $T^*(x,y)$ is the fundamental solution of the governing equation Eq. (1) and $q^*(x,y)$ is the derivative of $T^*(x,y)$, which are respectively expressed as

$$T^*(x,y)=\frac{1}{2\pi\sqrt{k_{11}k_{22}-k_{12}^2}}\ln\frac{1}{r} \tag{3}$$

$$q^*(x,y)=\frac{1}{2\pi\sqrt{k_{11}k_{22}-k_{12}^2}}\frac{(y_1-x_1)n_{x1}+(y_2-x_2)n_{x2}}{r^2} \tag{4}$$

In Eqs. (3) and (4), $n_{xi}(i=1,2)$ are the direction cosines of the outward normal n to the boundary Γ, r is the distance from the source point y to the field point x, which can be written as

$$r=\sqrt{s_{11}(y_1-x_1)^2+2s_{12}(y_1-x_1)(y_2-x_2)+s_{22}(y_2-x_2)^2} \tag{5}$$

in which $x_i(i=1,2)$ and $y_i(i=1,2)$ are the geometric coordinates of the field point x and source point y, s_{11}, s_{12} and s_{22} are defined by the heat conduction coefficients as follows:

$$s_{11}=k_{22}/(k_{11}k_{22}-k_{12}^2),\quad s_{12}=-k_{12}/(k_{11}k_{22}-k_{12}^2),$$
$$s_{22}=k_{11}/(k_{11}k_{22}-k_{12}^2) \tag{6}$$

The temperature and flux at an interior point y can be presented by the integral equations as

$$T(y)=\int_\Gamma T^*(x,y)q(x)d\Gamma-\int_\Gamma q^*(x,y)T(x)d\Gamma \tag{7}$$

$$q_{x_i}(y)=\int_\Gamma \frac{\partial T(x,y)^*}{\partial x_i}q(x)d\Gamma-\int_\Gamma \frac{\partial q(x,y)^*}{\partial x_i}T(x)d\Gamma \quad (i=1,2) \tag{8}$$

After the integrals are discretized along the boundary and the interpolating technique is introduced, Eq. (2) can be expressed in the matrix form as

$$[H]\{T\}=[G]\{q\} \tag{9}$$

where T and q are the vectors of temperature and flux on the boundary nodes, H and G are the corresponding coefficient matrices.

One way to model the coating-structure by the BEM is to use the multi-domain technique. In order to make this procedure clearer, only one layer of coating is considered here for the simplicity (see Fig. 1). The exterior boundary of the coating is Γ_1 and that of the substrate is Γ_2. The contact interface between the substrate and coating is Γ_I. Eq. (9) can be established respectively on the domain of the coating and substrate, which are presented as

$$\begin{bmatrix} H_1^C & H_I^C \end{bmatrix}\begin{Bmatrix} T_1^C \\ T_I^C \end{Bmatrix}=\begin{bmatrix} G_1^C & G_I^C \end{bmatrix}\begin{Bmatrix} q_1^C \\ q_I^C \end{Bmatrix} \tag{10}$$

$$\begin{bmatrix} H_I^S & H_2^S \end{bmatrix}\begin{Bmatrix} T_I^S \\ T_2^S \end{Bmatrix}=\begin{bmatrix} G_I^S & G_2^S \end{bmatrix}\begin{Bmatrix} q_I^S \\ q_2^S \end{Bmatrix} \tag{11}$$

in which, the superscript C and S refer to the coating and substrate, respectively, the subscript 1, 2 and I represent the boundary Γ_1, Γ_2 and Γ_I, respectively.

It is assumed here that the coating and the substrate are perfectly bonded to each other. The compatibility and equilibrium

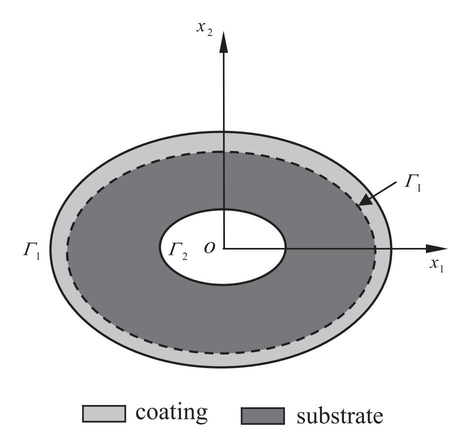

Fig. 1. A coating-structure with one layer coating.