Engineering Analysis with Boundary Elements 52 (2015) 1–15

Contents lists available at ScienceDirect

Engineering Analysis with Boundary Elements

journal homepage: www.elsevier.com/locate/enganabound

A meshless improved boundary distributed source method for two-phase flow monitoring using electrical resistance tomography

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ARTICLE INFO

Article history:
Received 7 July 2014
Received in revised form
5 November 2014
Accepted 17 November 2014
Available online 9 December 2014

Keywords:
Meshless methods
Improved boundary distributed source method
Electrical resistance tomography
Boundary estimation
Levenberg–Marquardt method

ABSTRACT

This paper presents a meshless method based on the improved boundary distributed source method (IBDS) to monitor two-phase flow in pipes using electrical resistance tomography (ERT). The conductivity of background liquid is assumed to be known *a priori* while the shape and location of the voids are the unknowns to be determined. The forward problem of ERT is solved using meshless IBDS method and the voids location and shape are reconstructed using Levenberg–Marquardt method. IBDS method is purely meshless and places its source and field points on the same physical boundary unlike conventional method of fundamental solution approach. Moreover, the elements of system matrix corresponding to Neumann and Dirichlet boundary conditions are evaluated analytically; therefore the IBDS method is computationally efficient that gives an accurate and stable solution. Numerical and experimental results with single and multiple voids are shown and the performance of IBDS method is compared with the boundary element method (BEM) in monitoring two-phase flow.

1. Introduction

Tomography applications in industry include locating the voids formed due to normal or accidental conditions inside the process vessel. Monitoring of such industrial process is essential for the safety and efficiency of mechanical equipment [1,2]. Electrical resistance tomography (ERT) is an imaging modality that reconstructs the internal resistivity distribution in the region of interest. The resistivity distribution is computed from the injected currents and measured voltages across the electrodes that are discretely attached on the outer surface of the process vessel [3]. The relationship between internal resistivity distribution and measured boundary voltages on the electrodes is nonlinear. Therefore, ERT forward problem is often solved using numerical methods, except for simple cases such as homogeneous or concentric anomaly case. Mesh based methods such as the finite element method (FEM) and the boundary based methods like the boundary element method (BEM), the finite difference method (FDM) are used to compute the ERT forward problem [4–6].

Although, FEM offers good reconstruction performance, its accuracy is dependent on the number of mesh elements used to compute the solution. For the works related to shape estimation in ERT employing FEM see [7–10]. To achieve better accuracy in shape reconstruction problem, FEM needs adaptive meshing or use of higher order approximation but this leads to increase in computational complexity [11]. BEM is better suited to shape estimation problems as compared to FEM as it discretizes the boundaries alone and the dimension of the problem is reduced by one [6,12–16]. Even though, BEM reduces the dimension of the problem by one, it still needs boundary discretization and also evaluation of singular boundary integrals to evaluate the solution. Especially, for higher dimensions, discretization of boundary for complex shaped object and evaluation of boundary integrals is not trivial. Moreover, for large number of discretized points on the boundary, the BEM system matrix becomes dense and is computationally intensive when compared to FEM. Recently, a new class of methods called meshless methods have gained considerable attention [17–22]. Meshless methods do not need any meshing or evaluation of boundary integrals and are mathematically simple and easy to use. Moreover, for less number of nodes, the computational time of meshless methods is less as compared to domain or boundary based methods. Another key advantage of meshless

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http://dx.doi.org/10.1016/j.enganabound.2014.11.008
0955-7997/

methods is that they can be easily extended to higher dimensions. In meshless methods, the approximation is built from nodes only, thus, meshless methods are particularly suitable for problems involving internal boundaries [17,20].

Many meshless methods have been proposed in the literature, among them, the method of fundamental solution (MFS) approach has gained attention among many engineering practitioners due to its ease in implementation. In the MFS, solution is approximated by a linear combination of fundamental solution across the nodes on the boundary satisfying the governing equation. [23–27]. In the MFS method, to avoid singularity of fundamental solution, a fictitious circle that lies interior or exterior to the physical boundary of the domain is considered for placing the source points. The distance at which this fictitious circle should be placed is still an open problem, especially, with irregular shaped boundary of domain. Generally, it is determined by experience or trial and error apporach hence the solution is not reliable. Newly developed methods such as the boundary collocation method [28,29], modified method of fundamental solution (MMFS) [30,31], singular boundary method (SBM) [32,33] and boundary distributed source method (BDS) [34] are proposed to overcome the drawback of the fictitious circle in MFS. The above methods place the source and field points on the same physical boundary and the singularity is handled in a different way. In the boundary collocation method, a nonsingular solution is chosen instead of the singular fundamental solution. In the MMFS and SBM, fundamental solution at singular points are replaced by origin intensity factor. The both methods differ in the way the origin intensity factor is evaluated. The MMFS applies numerical integration whereas in the SBM an inverse interpolation technique is used to evaluate the origin intensity factor [33,35]. A distributed source across each source point is considered in BDS and the singular fundamental solution is integrated over the areas of the distributed source [34,36].

In SBM, to evaluate the origin intensity factor, few sample nodes placed in the interior or exterior to the physical boundary are needed. The location of these sample nodes have an effect on the solution accuracy of SBM. An improved formulation of SBM is proposed in [33] that avoids these sample nodes in evaluating the origin intensity factor. It uses an inverse interpolation method and substracting and adding back technique to evaluate the origin intensity factor. The improved formulation has an analytic expression for evaluating the diagonal terms of fundamental solution derivatives (Neumann problem). However, in the case of diagonal terms for fundamental solution (Dirichlet problem), it uses an indirect approach using inverse interpolation method. In the BDS method, an analytic expression to compute the diagonal terms of fundamental solution for a circular distributed source is easily derived. But, to compute the diagonal terms of derivatives of fundamental solution, an indirect method proposed by Sarler [37,38] is used. This indirect method of evaluating diagonal elements leads to increase in computational time and moreover the solution is not reliable. Kim [39] proposed an improved boundary distributed source (IBDS) method, where an analytic expression for diagonal elements of Neumann boundary conditions is derived by considering the fact that the boundary integration of the normal gradient of potential should vanish. IBDS method thus has analytic expressions for determining the diagonal elements of both Neumann and Dirichlet conditions. The IBDS solution is stable with good accuracy moreover it is computationally efficient due to the elimination of indirect method to evaluate diagonal elements [39]. IBDS method is formulated to solve ERT forward problem using complete electrode model [40] and its solution is compared against FEM and BEM.

In this paper, IBDS method is applied to visualize two phase flow monitoring to locate voids or cavities that appear in process vessel using electrical resistance tomography. The conductivity of background substance is assumed to be known *a priori* while the location and shape of void are the unknowns to be detemined. The boundary of void is parameterized using truncated Fourier series and the coefficients of Fourier series are estimated using Levenberg–Marquardt method. The IBDS method is truly meshless and evaluates the diagonal terms corresponding to Dirichlet and Neumann boundary conditions analytically. Therefore, it is computationally efficient and the forward solution is stable. The proposed method is tested with numerical simulations and phantom experiments and the reconstruction performance is compared against BEM. The results show a promising peformance of meshless IBDS method with LM for the identification of voids or cavities.

The reminder of the article is organized as follows. In Section 2, at first, a mathematical model of ERT physical model is described and then forward problem is formulated using meshless IBDS method. At last, in Section 2, boundary parameterization of cavities is described. The inverse problem of estimating the boundary parameters of void is given in Section 3. In Section 4, we present the results of reconstructing the boundaries of cavities using numerical simulations and laboratory experiments. Finally, concluding remarks are given in Section 5.

2. ERT forward problem and IBDS method

2.1. Mathematical description of ERT physical model

ERT has L discrete electrodes $e_l(l=1,2,\dots,L)$ attached on the boundary $\partial\Omega$ of industrial process vessel. Currents of magnitude $I_l(l=1,2,\dots,L)$ are injected through the electrodes into the domain Ω that comprises of substances from the industrial process and the resultant excited voltages are measured over the surface of the electrodes. The relationship between the internal conductivity distribution σ and the electrical potential u on Ω is governed by a partial differential equation derived from Maxwell equations, i.e. [3]

$$\nabla\cdot\sigma(p)\nabla u(p)=0, p=(x,y)\in\Omega \tag{1}$$

where p refers to the spatial location (x, y) within the domain Ω.

Let us consider two-phase flow inside pipeline or process vessel such that a void appears in the flow domain Ω surrounded by the background liquid. The void occupies region D in the flow domain Ω and has conductivity σ_a embedded in the homogeneous background liquid with conductivity σ_b (Fig. 1). The conductivity distribution inside the flow domain Ω can be represented as

$$\sigma(p)=\sigma_b+(\sigma_a-\sigma_b)\chi_D(p) \tag{2}$$

where $\chi_D(p)$ is defined as a characteristic function that has a value of 1 within the region D and zero otherwise. If the conductivities are assumed to be known *a priori* as in the case of two-phase flow, the conductivity estimation problem is transformed into a shape estimation problem [8]. If the conductivities of void and background are assumed to be constant then the governing equation

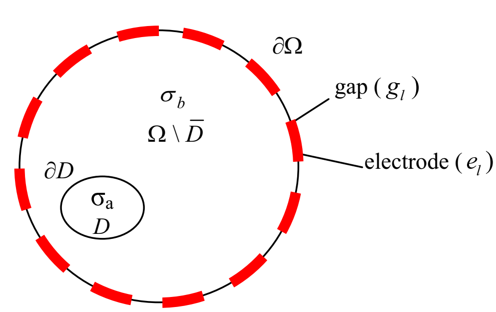

Fig. 1. Process vessel with electrodes attached over its periphery to estimate the shape and location of void embedded within the homogeneous background.