Engineering Analysis with Boundary Elements 52 (2015) 16–23

Contents lists available at ScienceDirect

Engineering Analysis with Boundary Elements

journal homepage: www.elsevier.com/locate/enganabound

Cracked plate analysis with the dual boundary element method and Williams' eigenexpansion

J. Caicedo, A. Portela*

Department of Civil Engineering, Faculty of Technology, University of Brasília, Brasília-DF 70910-900, Brazil

ARTICLE INFO

Article history:
Received 21 June 2014
Received in revised form
13 October 2014
Accepted 19 November 2014
Available online 12 December 2014

Keywords:
Cracked plates
Williams' eigenexpansion
Singularity subtraction technique
Dual boundary element method
J-integral
Stress intensity factors

ABSTRACT

This paper provides a numerical verification that the singular term of Williams' series eigenexpansion can be used as a singular solution, valid in the neighborhood of each crack tip, in a single-region dual boundary element analysis of two-dimensional piece-wise flat multi-cracked plates, either with edge or internal cracks, in mixed-mode deformation, as an intermediate and necessary research step towards the implementation of the singularity subtraction technique.

The dual equations are the displacement and traction boundary integral equations which allow the solution of general mixed-mode crack problems in a single-region boundary-element analysis.

The singularity subtraction technique is a regularization procedure that uses a singular particular solution of the crack problem to introduce the stress intensity factors as additional primary unknowns in the dual boundary element method. Its implementation depends on the availability of closed-form singular solutions relative to a single-region of a general multi-cracked plate.

In this paper, Williams' series eigenexpansion, which is valid for a semi-infinite edge crack, is used to compute the stress intensity factors, for both cases of edge and internal cracks, for each deformation mode. The singular term of the expansion is used as a singular particular solution in the neighborhood of each edge and internal crack tip. Collocation of this term, at a single internal point near the crack tip, is carried out to compute the stress intensity factors in post-processing.

Several cracked plates were analyzed with this technique in order to assess the validity of using the singular term of Williams' series eigenexpansion for the regularization of the elastic field in a single-region dual boundary element analysis of a general piece-wise multi-cracked plate. The results obtained in this work are in perfect agreement with those obtained with the dual boundary element method, through the J-integral technique, and other published results for both cases of the edge and internal piecewise-flat cracks. Hence, it can be concluded that, in the singularity subtraction technique of the dual boundary element analysis of general edge and internal piecewise-flat multi-cracked plates under mixed-mode deformation, the singular term of Williams' series can be used as a closed-form particular solution, valid in the neighborhood of each crack tip.

1. Introduction

The boundary element method is a well-established numerical technique in the engineering community, see Brebbia [1] and Brebbia et al. [2]. The boundary element method has been successfully applied to linear elastic problems in domains containing no degenerated geometries. These degeneracies, either internal or edge surfaces which include no area or volume and across which the problem field is discontinuous, are defined as mathematical cracks. For symmetric crack problems only one side of the crack needs to be modelled and a single-region boundary element analysis may be used. However, in a single-region analysis, the solution of general crack problems cannot be achieved with the direct application of the boundary element method, because the coincidence of the crack boundaries causes an ill-posed problem. Effectively, for a pair of coincident source points on crack boundaries, the algebraic equations relative to one of the points are identical with those equations relative to the opposite point, since the same boundary integral equation is applied at both coincident source points, with the same integration path, around the whole boundary of the problem. Among the techniques devised to overcome this difficulty, the most general are the sub-regions method, presented by Blandford et al. [7] and the dual boundary-element method (DBEM), first presented by Portela et al. [8] in elasticity. The main drawback of the method of subregions is that the introduction of

* Corresponding author. Tel.: +55 61 8154 7262.
E-mail address: aportela@unb.br (A. Portela).

http://dx.doi.org/10.1016/j.enganabound.2014.11.010

artificial boundaries, which connect the cracks to the boundary so that the domain is partitioned into subregions without cracks, is not unique and thus it cannot be easily implemented into an automatic procedure.

On the other hand, the DBEM is the most efficient technique devised to overcome this difficulty. It introduces two independent equations, the displacement and traction boundary integral equations, with the displacement equation applied on one of the crack surfaces and the traction equation on the other. As a consequence, general mixed-mode crack problems can be solved in a single-region boundary element formulation, with both crack surfaces discretized with boundary elements. The use of dual integral equations was first reported by Bueckner [3], in crack problems, and by Watson [4], in the boundary element method. The theoretical bases of the DBEM were presented by Hong and Chen [5]. A thorough review article of dual boundary element methods was presented by Chen and Hong [5].

Within the limits of linear elastic analysis, the stress field is unbounded at a crack tip. This was early reported by Brahtz [11] and later by Williams [12] who after an investigation of the analytical form of these singularities demonstrated that under all possible combinations of boundary conditions, the stress becomes infinite at the tip of a crack. From a physical point of view, unbounded elastic fields are meaningless. Nevertheless, unbounded stresses cannot be ignored as their presence indicates that new phenomena (e.g. plasticity and fracture) may occur leading to localized damage in practical situations. In this paper, the term singularity is used to denote the cases in which the elastic stress field becomes unbounded. If r denotes the distance measured from the crack tip, the stress field is of the order $r^{-1/2}$ which becomes singular as r tends to zero. The stress intensity factor, defined at the crack tip, is a measure of the strength of this singularity.

In the DBEM, the computation of the stress intensity factors was first carried out in post-processing using the J-integral technique, as reported by Portela et al. [8]. Although this technique does not perform a regularization of the elastic field, it is very accurate because it uses the elastic field computed at internal points which is a highly accurate operation in the boundary element method since the exact variation of the interior elastic field is built into the fundamental solutions. Despite its high accuracy, the J-integral technique tends to become very expensive when a high number of internal points are considered in the path of the J-integral.

The singularity subtraction technique was presented by Portela et al. [9], as an alternative to the J-integral technique. The singularity subtraction technique is a regularization procedure that uses a singular particular solution of the crack problem to introduce the stress intensity factors as additional primary unknowns in the DBEM. This technique is extremely accurate since the crack-tip singularities are not present in the DBEM analysis of the regularized elastic field. Despite its high accuracy, a possible drawback of the singularity subtraction technique is the lack of available closed-form singular solutions valid in a single-region boundary element analysis. Effectively, in many practical problems the path of a crack, although curved, is usually modelled as piece-wise flat. For an arbitrary piece-wise flat crack there is not a closed-form particular solution of the problem, valid in the global domain, for use in the regularization of the singularity subtraction technique of the problem.

Hence, in order to overcome the referred difficulties, there is a clear need of research in this field to make the singularity subtraction technique a robust and efficient numerical method for the boundary element analysis of general multi-cracked plates.

In this paper, the first term of Williams' [13] series eigenexpansion, which represents the singular elastic field around the crack tip of a semi-infinite crack, is used in the neighborhood of each crack tip, for both cases of edge-crack and internal-crack problems, as well as for the case of piece-wise flat crack problems. This is a key point of this research, since it is well known that, for a finite flat internal crack, the singular particular solution representing the elastic field around the two crack tips can be defined in terms of Westergaard [14] stress functions which are valid for a finite flat crack in an infinite plate and, for an arbitrary piece-wise flat crack, there is not a closed-form particular solution of the problem.

Collocation of the singular term of Williams' series eigenexpansion, at an internal point approaching each crack tip, implemented in post-processing, seems to be the simplest way of assessing the validity of the procedure of computing the stress intensity factors in the singularity subtraction technique of a general piece-wise flat multi-crack problem. This internal-point procedure extends, to the analysis of cracked plates, a similar approach implemented by Portela et al. [15], in the analysis of notched plates.

Therefore the main focus of this paper is simply to provide a numerical verification that the singular term of Williams' series eigenexpansion can be used as a singular solution, valid in the neighborhood of each crack tip, in a single-region dual boundary element analysis of two-dimensional piece-wise flat multi-cracked plates, either with edge or internal cracks, in mixed-mode deformation. Rather than presenting a new technique to compute stress intensity factors, this approach is thus regarded only as a simple and necessary intermediate research step for the future implementation of the singularity subtraction technique.

The organization of the paper is as follows. After the introduction, the dual boundary integral equations are summarized in Section 2. The computation of the stress intensity factors is presented in Section 3, where the Williams' field is introduced, the normal and shearing stress components at an internal point are presented and the analysis methodology is defined. In Section 4, some numerical results are presented illustrating the effectiveness of the present analysis procedure. Finally, the concluding remarks and future developments are presented.

2. The dual boundary element method

The DBEM performs the analysis of general crack problems in a single-region boundary element formulation, as represented in Fig. 1. The equations on which the DBEM is based are the displacement and the traction Somiglianas boundary integral equations, as presented by Portela et al. [8]. In the absence of body forces and assuming continuity of the displacements at a boundary point P, the boundary integral representation of the displacement components u_i is given by

$$c_{ij}(P)u_j(P) + \fint_{\Gamma} T_{ij}(P,Q)u_j(Q)\, dS(Q) = \int_{\Gamma} U_{ij}(P,Q)t_j(Q)\, dS(Q), \tag{1}$$

where i and j denote Cartesian components; $T_{ij}(P,Q)$ and $U_{ij}(P,Q)$ represent, respectively, the traction and displacement Kelvin fundamental solutions, at a boundary point Q; the symbol $\fint_{\Gamma}$ stands for Cauchy principal-value integral, and the coefficient $c_{ij}(P)$ is given by $\frac{1}{2}\delta_{ij}$ for a smooth boundary at the point P in which δ_{ij} is the

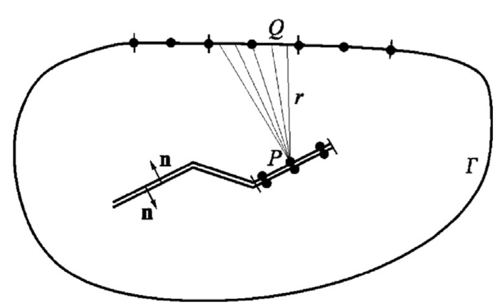

Fig. 1. Single-region analysis of DBEM.