Engineering Analysis with Boundary Elements 52 (2015) 44–55

Contents lists available at ScienceDirect

Engineering Analysis with Boundary Elements

journal homepage: www.elsevier.com/locate/enganabound

Properties of integral operators and solutions for complex variable boundary integral equation in plane elasticity for multiply connected regions

Y.Z. Chen *, Z.X. Wang

Division of Engineering Mechanics, Jiangsu University, Zhenjiang, Jiangsu 212013, People's Republic of China

ARTICLE INFO

Article history:
Received 23 July 2014
Received in revised form
7 September 2014
Accepted 19 November 2014
Available online 17 December 2014

Keywords:
Complex variable boundary integral equation
The domain field equality
The null field BIE
Plane elasticity
Properties of integral operators and solutions for CVBIE

ABSTRACT

This paper studies properties of integral operators and solutions for CVBIE (complex variable boundary integral equation) in plane elasticity for multiply connected regions. Four cases for considered regions are studied. For the individual case, we study (a) the domain field equality, (b) the null field BIE and (c) the usual CVBIE. Properties of integral operators or the kernels are studied in detail, which is based on the properties of Cauchy type integral. The Neumann problem is considered. It is shown that for finite region cases (Sections 2 and 3) the CVBIE allow three modes of rigid motion along contours under the traction free condition. In addition, for infinite region cases (Sections 4 and 5) the CVBIE does not allow three modes of rigid motion.

© 2014 Elsevier Ltd. All rights reserved.

1. Introduction

The boundary element method (BEM) is a numerical technique based on boundary integral equation (BIE), which was developed by some pioneer researchers [1–4]. Comparing with the finite element method, the BEM has a well-known dimensionality advantage. Therefore, the numerical procedures based on BIE become the third important technique in the numerical analysis of elasticity problem [5].

In the BIE, there are two kinds of formulation, for example, in plane elasticity. One is the direct BIE method, and other is the indirect BIE method [5,6]. In the direct BIE method, the unknown functions are the displacements and tractions along the boundary. However, in the indirect BIE method the unknown function is an intermediate function. Since both methods reflect the nature of the governing equation, for example, the Laplace equation, both methods can be used to solve the boundary value problem (BVP). For the boundary value problem of Laplace equation, the direct and indirect BIE methods were summarized [5].

Generally, the Somigliana identity is used in the direct method of BIE for elasticity problem [4,5]. Clearly, the Somigliana identity is a result of usage of Betti's reciprocal theorem between the fundamental field and the physical field.

However, the BIE formulations for elasticity problem suffer some inconvenient points. For example, at a particular size of domain for the Dirichlet boundary value problem in plane elasticity, a displacement-stress field can be found in the studied region even vanishing displacement is assumed along the boundary. The mentioned problem is called the degenerate scale problem in BIE. Many researchers studied this problem [7–9]. Clearly, the degenerate scale represents an illness condition and researchers must avoid using the degenerate scale in real computation.

One more important topic in plane elasticity is uniqueness of solution for displacement in the Neumann boundary value problem. To this end, one must study some operators acted upon the displacement in detail.

Based on a general, operational approach, two new integral identities for the fundamental solutions of the potential and elastostatic problems were established in the paper [10]. Non-singular forms of the conventional boundary integral equations (BIEs) are derived by employing these two identities for the fundamental solutions and the two terms subtraction technique. The non-singular nature of the boundary integral equations (BIEs) in the boundary element method (BEM) was discussed in the paper [11]. After some substitution, one can arrive at a weak singular of the BIE in plane elasticity. Four integral identities

* Corresponding author. Tel.: +86 511 88780780; fax: +86 511 88791739.
E-mail address: chens@ujs.edu.cn (Y.Z. Chen).

http://dx.doi.org/10.1016/j.enganabound.2014.11.009
0955-7997/© 2014 Elsevier Ltd. All rights reserved.

satisfied by the fundamental solution for elastostatic problems are reviewed and slightly different forms of the third and fourth identities were presented [12].

A new kernel with formulation of the relevant BIE in plane elasticity was introduced [13,14]. The new kernel is derived from a fundamental solution expressed in a pure deformable form (see Appendix A). If the kernel is used, the regularity condition at infinity is satisfied for any loadings applied on the contours. The derivation for the domain field equality was based on the Somigliana identity in the complex variable form [15]. The generalized Sokhotski–Plemelj's formulae are used to obtain the CVBIE. Properties of integral operators in CVBIE in plane elasticity are studied. The regularity condition at infinity in the exterior boundary value problem of plane elasticity was studied [16]. It is found that the usual suggested kernel does not satisfy the regularity condition at infinity when the loadings on contours are not in equilibrium. A new kernel from a revised displacement expression in the fundamental field is suggested, which satisfies the regularity condition at infinity in the general loading case along the contours. Recently, an approach is suggested to study the regularization of the non-unique solution [17].

This paper studies properties of integral operators and solutions for CVBIE in plane elasticity for multiply connected regions. The following four particular cases are studied: (a) boundary value problem for an interior region (in Section 2), (b) boundary value problem for a finite multiply connected region (in Section 3), (c) boundary value problem for an exterior region (in Section 4) and (d) boundary value problem for an infinite multiply connected region (in Section 5). For the individual case, we study (a) the domain field equality, (b) the null field BIE and (c) the usual CVBIE. Properties of integral operators or the kernels are studied in detail, which is based on the properties of Cauchy type integral. The Neumann problem is considered. It is shown that for finite region cases (Sections 2 and 3) the CVBIE allows three modes of rigid motion along contours under the traction free condition along contours. In addition, for infinite region cases (Sections 4 and 5) the CVBIE does not allow three modes of rigid motion along contours under the traction free condition along contours.

2. Formulation and properties of solutions for CVBIE for the interior region

The way for the formulation of CVBIE for the interior region is introduced (Fig. 1). Prosperities for kernels and solutions for of CVBIE for the interior region are analyzed in detail.

2.1. Some preliminary knowledge in complex variable method of plane elasticity

The complex variable function method plays an important role in plane elasticity. Fundamental of this method is introduced. In the method, the stresses $(\sigma_x, \sigma_y, \sigma_{xy})$, the resultant forces (X, Y) and the displacements (u, v) are expressed in terms of complex potentials $\varphi(z)$ and $\psi(z)$ such that [18]

$$\sigma_x + \sigma_y = 4\mathrm{Re}\Phi(z),$$
$$\sigma_y - \sigma_x + 2i\sigma_{xy} = 2[\bar{z}\Phi'(z) + \Psi(z)] \tag{1}$$

$$f = -Y + iX = \varphi(z) + z\overline{\varphi'(z)} + \overline{\psi(z)} \tag{2}$$

$$2G(u+iv) = \kappa\varphi(z) - z\overline{\varphi'(z)} - \overline{\psi(z)} \tag{3}$$

where $\Phi(z) = \varphi'(z)$, $\Psi(z) = \psi'(z)$, a bar over a function denotes the conjugated value for the function, G is the shear modulus of elasticity, $\kappa = (3-\nu)/(1+\nu)$ in the plane stress problem, $\kappa = 3-4\nu$ in the plane strain problem, and ν is the Poisson's ratio. Sometimes, the displacements u and v are denoted by u_1 and u_2, the stresses σ_x,σ_y and σ_{xy} by σ_1,σ_2 and σ_{12}, the coordinates x and y by x_1 and x_2.

Except for the physical quantities mentioned above, from Eqs. (2) and (3) two derivatives in specified direction (abbreviated as DISD) are introduced as follows [19,20]:

$$J_1(z) = \frac{d}{dz}\{-Y+iX\} = \Phi(z) + \overline{\Phi(z)} + \frac{d\bar{z}}{dz}(z\overline{\Phi'(z)} + \overline{\Psi(z)}) = \sigma_N + i\sigma_{NT} \tag{4}$$

$$J_2(z) = 2G\frac{d}{dz}\{u+iv\} = \kappa\Phi(z) - \overline{\Phi(z)} - \frac{d\bar{z}}{dz}(z\overline{\Phi'(z)} + \overline{\Psi(z)})$$
$$= (\kappa+1)\Phi(z) - J_1 \tag{5}$$

It is easy to verify that $J_1 = \sigma_N + i\sigma_{NT}$ denotes the normal and shear tractions along the segment $\overline{z, z+dz}$ (Fig. 1). Secondly, the J_1 and J_2 values depend not only on the position of a point "z", but also on the direction of the segment "$d\bar{z}/dz$".

2.2. Formulation of CVBIE for the interior region

In the following analysis, the α-field shown by Fig. 1(a) is relating to the fundamental field caused by concentrated force at the point $z = \tau$. The relevant complex potentials are as follows [18]:

$$\varphi(z) = F\ln(z-\tau), \varphi'(z) = \Phi(z) = \frac{F}{z-\tau}, \varphi''(z) = -\frac{F}{(z-\tau)^2} \tag{6}$$

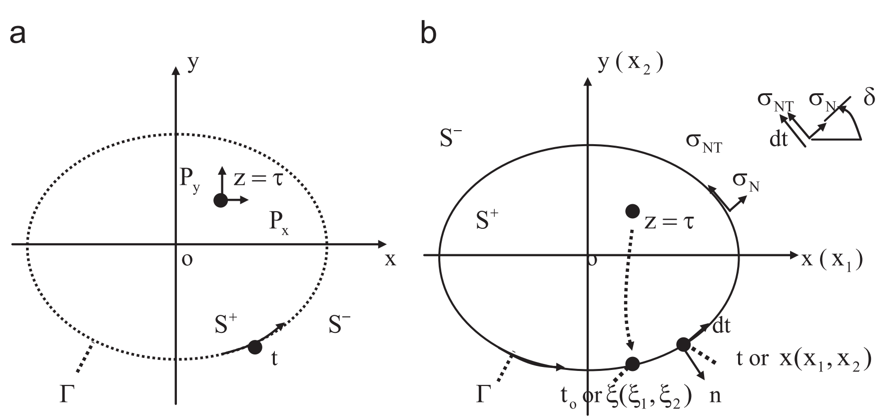

Fig. 1. (a) The α-field with the concentrated forces applied at $z = \tau$, (b) the β-field, or the physical field defined on a finite region.

Download English Version:

https://daneshyari.com/en/article/512378

Download Persian Version:

https://daneshyari.com/article/512378

Daneshyari.com