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Nonsingular boundary element flexural analysis of stiffened plates

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ABSTRACT

This paper concerns a nonsingular formulation to deal with thin plates reinforced by beams using a BEM-FEM combination. The nonsingular formulation developed for the Kirchhoff's plate bending problem is extended to include the effect of the stiffener beams. In the proposed model, the plate is modelled by BEM using quadratic elements with transverse deflection and rotations in two orthogonal directions as the degrees of freedom (w, w_x and w_y). The beam is modelled by FEM using Timoshenko beam theory with quadratic elements and taking into account the transverse shear and bending and twisting moments (three degrees of freedom per node). As the plate and beam are modelled using C^0 quadratic interpolation with the same set of three nodal values, these can be easily coupled. The nonsingular BEM formulation for thin plate bending problems is presented and the coupling of the BE equations of the plate and the FE equations of the beam is done by using the compatibility and equilibrium equations. A computer program has been developed using C++ with object oriented programming concepts. Several numerical examples are presented to illustrate the accuracy and efficiency of the present formulation.

1. Introduction

The analysis of plates and shells in bending is a problem of great significance in several branches of engineering such as aerospace, mechanical and structural engineering. The structural plate systems reinforced by beams is commonly used in many structures which require high strength combined with light weight such as bridge decks, building floors, modern industrial structures, ship hulls and deck, aircraft fuselage and wings, automobile body, and in myriad other applications. As a result, the flexural analysis of stiffened plates continues to be a subject of great interest. Several methods have been suggested to solve the problem using a variety of approaches. A comprehensive literature review of stiffened plate can be found in the review paper by Satsangi and Mukhopadhay [1]. Early research concerning the stiffened plate bending analysis was based on various assumptions and approximations as there are many complications in analysing and predicting the behaviour and a complete understanding of its behaviour is not fully realised even now. Initially the orthotropic plate theory was used which replaced the stiffened plate as a plate of constant thickness with the stiffener as an additional smeared layer, and the plate was considered to be having different rigidities in the two orthogonal directions. This approach yielded satisfactory results only when identical stiffeners were provided at close spacing. Another theory was the grillage theory in which the effects of the plates were included into the stiffener by increasing the second moment of area of the beam. This model, though simple, could not accurately evaluate the stresses in the plate and the stiffener. To improve the situation, subsequent researchers developed three differential equations for the plate and later transformed the three differential equations into one eighth order partial differential equation for deflection. These theoretical approaches involved tedious computational procedures for the evaluation of deflection and stresses, and were not suitable for analysing practical problems involving plates of arbitrary geometry and irregularly placed stiffeners. Several numerical methods such as finite difference method (FDM), finite element method (FEM) and boundary element method (BEM) have been used for the flexural analysis of stiffened plates. A combination of FEM and BEM has also been attempted successfully. Gustafson and Wright [2] were the first to apply FEM to the solution of stiffened plates. The plate was idealised using four-noded parallelogram plate elements and stiffeners modelled using two-noded beam elements. McBean [3] documented the mathematical derivations of the governing differential equations and the associated boundary conditions for FEM using the principle of minimum total potential energy. A stiffener element compatible with the quadrilateral plate element was derived and solution based on this combination of elements provided a lower bound on the strain energy. Since then a substantial amount of work on finite element analysis of stiffened plates has been carried out

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using several modelling strategies, which mainly differ on the assumptions related to displacements of stiffeners. However, it was soon realised that the use of FEM for solving stiffened plate problem requires significant computer storage and time to model the plate-beam system if the stiffeners are not equally spaced or have different geometric properties. Furthermore, the FEM requires that the stiffeners be placed along the mesh lines, and thus for any change in the position of stiffener remeshing of the plate becomes inevitable.

BEM has already been established as an efficient and accurate computational tool for solving a large number of complex engineering problems. The internal stress concentrations due to loads distributed over small regions can be obtained with high precision in this method. In addition, the shear force is evaluated accurately when compared with other numerical methods because the shear force is not obtained by differentiating the interpolation function as for other numerical techniques. Moreover, the order of error is same in computing deflections, slopes, moments and shear forces. Many formulations for the boundary element analysis of thin plate bending problems have been proposed over the years. Several BEM formulations have been proposed by various researchers for the analysis of stiffened plate bending problems. These can be broadly classified as the BEM–FEM coupling [4–10], subregion technique [11–13], method of zoned plates [19–21], and BEM based methods [14–18,22–25]. BEM coupled with FEM appears as the natural numerical procedure to analyse plates stiffened by beams, where the BEM is used to model the comparatively large area of plate and FEM is used to approximate the slender beam. The coupling is achieved by ensuring compatibility and equilibrium. Practical applications of BEM–FEM coupling for modelling stress distributions in other engineering problems can be found in the work of Ameen et al. [4], Ameen [5], Beer [6], Coda and Venturini [7], Coda et al. [8], Coda [9] and Ng et al. [10]. Ng et al. studied slab-beam bridges using the combined BEM–FEM technique. The slab was modelled by BEM and the stiffener beam by FEM assuming only vertical internal forces to develop between the two neglecting torsion. Venturini [11] used the classic subregion technique in which the boundary integrals were solved analytically, whether or not they included singularities. Leite et al. [12] presented the subregion technique in which equilibrium is preserved along the interface without traction approximation, requiring only displacement along the interface and the subregion assumed to be thin to simulate stiffeners.

Early analyses of stiffened plates by BEM mainly focussed building floor slabs. The earliest application of BEM for building slab analysis was presented by Bezine [13]. He derived the plate equations with internal boundary conditions which could be used to simulate rigid columns connected to the plate. A formulation for coupling plates with beams and flexible columns was presented by Paiva and Venturini [14,15]. Hartley et al. [16] presented a plate bending formulation which could include rigid patches in the domain and could be used for building slab analysis. An alternative BEM formulation of plate coupled with beam and column with three nodal degrees of freedom was provided by Paiva [17]. In all these works, the plate thickness was considered uniform. The thickness variation can be considered using the subregion formulation in which each subregion is analysed separately and then joined together considering equilibrium and compatibility conditions [18,19]. This leads to a large system of equations with blocks of zero elements in the matrices. Venturini and Paiva [18] proposed a scheme to deal with zoned domains without dividing them into subregions, considering the displacements along the interface as the only variable. However, singularity at the interface was a problem for obtaining the internal tractions along the interface and also had difficulty in modelling corners where more than one subregion met, and so were forced to use discontinuous elements. An alternative formulation for zoned plate bending was proposed by Paiva and Aliabadi [19] in which the floor was analysed as a single structural member and compatibility and equilibrium conditions were not required. Here, the boundary equations for the corners were obtained easily and resulted in a smaller system of equations compared to the subregion technique. Fernandez and Venturini [20] extended this method to stiffened plates without dividing it into beam and plate elements and treating beams as regions of different thickness which can be very narrow thereby representing the stiffness variation. The stiffener rigidity influence included by two line integrals along the beam sides and quasi-singular integral scheme was used to obtain the algebraic equations. Fernandes et al. [21] presented new applications of this formulation.

A direct boundary element method for the static bending analysis of plates stiffened by beams of open or closed cross section was presented by Tanaka and Bercin [22]. They considered the bending, torsional and warping rigidities and eccentricity of the stiffeners. A boundary element analysis for the shear deformable stiffened plate was presented by Wen et al. [23] based on the shear deformable plate elastostatic theory. The rotations and deflection for the shear deformable plate and displacements for two dimensional plane stress elasticity were determined solving the boundary integral equation numerically. An improved model for the analysis of plates stiffened by parallel beams of doubly symmetric cross section with deformable connexions was presented by Sapountzakis and Mokos [24], in which the stiffening beam was isolated from the plate by sections in the lower outer surface of the plate so that the plate and the beams can slip in all directions of the connection without separation and also taking into account the tractions arising in all directions at the fictitious interfaces. This model takes into account the non-uniform distribution of interface transverse shear and torsional forces. A boundary element formulation for the stiffened plate bending considering line distributed interactive forces and moments approximated by suitable family of interpolation functions was presented by Tanaka and Oida [25].

This paper presents a general nonsingular formulation for the bending analysis of thin plates stiffened by beams. The nonsingular formulation developed for thin plate bending [26] is extended here. The displacement and gradient boundary integral equations are regularised by applying a constant displacement field and a constant displacement gradient field respectively. The structure is idealised as assembled plate and stiffener elements, rigidly connected at the junction. The plate is modelled using BEM whereas FEM is used to model the beam. Quadratic elements with C^0 continuity having three degrees of freedom per node (transverse deflection and rotations in two normal directions) are used. The stiffeners are modelled using Timoshenko beam elements as it is a C^0 quadratic element with three degrees of freedom per node. Thus, both the BEM formulation and Timoshenko beam element share the same type and number of degrees of freedom at each node where the stiffener joins the plate. Transverse shear, bending moment and twisting moments are considered to act as line-distributed generalised forces in the plate and the beam along the interface. The coupling of BEM and FEM equations is achieved by ensuring the compatibility of the displacements and establishing equilibrium between the forces acting at the interface of the plate and the beam. The system of BEM and FEM equations are expressed in terms of different variables and cannot be combined directly. The coupling of the two sets of equations has been studied by various researchers and different approaches such as the FEM hosted approach, BEM hosted approach and the iterative domain decomposition method are in use. Here, the coupling of BEM and FEM is done by a BEM hosted approach of Ameen et al. [4,5] which introduces no approximation and is better suited for cases in which the boundary element matrices are much larger compared to the finite element matrices. To demonstrate the accuracy and validity of the proposed method a number of numerical examples of both ribbed and grid plate systems are presented and the results are compared with the known results in literature.