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A meshless local natural neighbour interpolation method to modeling of functionally graded viscoelastic materials

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ABSTRACT

A meshless local natural neighbour interpolation (MLNNI) method applied to solve two-dimensional quasi-static and transient dynamic problems in continuously heterogeneous and linear viscoelastic media is presented and discussed in this article. The analysis is performed using the correspondence principle and the Laplace transform technique. In the present method, nodal points are spread on the analyzed domain and each node is surrounded by a polygonal sub-domain. The trial functions are constructed by the natural neighbour interpolation and the three-node triangular FEM shape functions are taken as test functions. The natural neighbour interpolants are strictly linear between adjacent nodes on the boundary of the convex hull. As a result, the essential boundary conditions can be imposed by directly substituting the corresponding terms in the system of equations. To get the viscoelastic solution in the time domain, the inverse Laplace transform algorithm of Stehfest is employed. Some numerical examples are given at the end to demonstrate the availability and accuracy of the present method.

1. Introduction

The interest in functionally graded materials (FGMs) has been steadily growing in recent years due to their continuously and smoothly varying material properties. If properly designed, FGMs can offer various advantages such as reduction of thermal stresses, minimization of stress concentration or intensity factors and attenuation of stress waves [1]. Therefore, they have gained potential applications in a wide range of engineering structures and components such as electronic devices, corrosion-resistant and wear-resistant coatings, thermal barrier coatings and biomaterials [2,3]. Therefore, the behavior of FGMs needs to be better understood to fully exploit their characteristics. So far, extensive research has been carried out to efficiently and accurately simulate FGMs. Some representative examples are the works by Zhang and Paulino [4], Arciniega and Reddy [5], Song and Paulino [6], Sladek et al. [7], Wang and Qin [8] and so on. However, most of the previous research has been limited to elastic material behavior. As a matter of fact, FGMs exhibit creep and relaxation behavior [9] and therefore accurate simulation of them necessitates the use of viscoelastic constitutive models. In the framework of linear continuum theory, such a behavior can be described by linear viscoelasticity. Generally speaking, the correspondence principle [10] is one of the most useful tools in viscoelasticity because the Laplace transform of a viscoelastic solution can be directly obtained from the corresponding elastic solution. After solving the transformed problem, the solutions are transformed back to the time domain by numerical methods. Unfortunately, for general linear viscoelastic FGMs, the correspondence principle may not be applicable. To avoid this problem, Paulino and Jin [11] proved that the correspondence principle may be used to obtain viscoelastic solutions only for a class of FGMs when the relaxation moduli are separable in space and time. Based on such particular correspondence principle for FGMs, Paulino and Jin [12,13] have subsequently studied crack problems of FGM strips subjected to antiplane shear loading and later to in-plane loading [14]. Besides, Sladek et al. [15] presented a meshless local Petrov-Galerkin method for static and dynamic analysis of continuously nonhomogeneous and linear viscoelastic solids. In the article, the transformed meshless formulations for solving nonhomogeneous viscoelastic problems are very similar to those for solving nonhomogeneous elastic problems.

Although the finite element method (FEM) [16,17] and the boundary element method (BEM) [18] have been successfully established and applied to a variety of engineering problems, there is still a growing interest in developing new advanced numerical methods [19–24]. In recent years, meshless methods have been drawing much attention from researchers due to their higher adaptivity and lower cost in preparing input data for numerical analysis. As a fundamental base for the derivation of many meshless formulations, the meshless local Petrov-Galerkin (MLPG) method [25] differs significantly with other meshless methods in that the governing equation is satisfied node-by-node in a local way of making weighted residual zero over a sub-domain. The MLPG method provides the flexibility in choosing

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the trial and test functions, as well as the sizes and shapes of local sub-domains and therefore brings forward a series of new methods [26–31] with amazing flexibility and efficiency. Among these, the meshless local natural neighbour interpolation (MLNNI) method [30,31] is formulated to combine the advantage of easy imposition of essential boundary conditions of natural element method (NEM) [32] with some prominent features of the MLPG method. Remarkable successes of the MLNNI method have been reported in a wide range of computational problems [33–37].

In this article, the MLNNI method is applied to solve linear viscoelastic problems of FGMs by incorporating the correspondence principle. In our meshless method, a set of properly scattered nodes within the problem domain is employed to represent the continuous problem and no elements or meshes are required. Each node is surrounded by a polygonal sub-domain, which can be easily constructed with Delaunay tessellations. The viscoelastic solids can be effectively treated in the Laplace domain by means of the MLNNI method. Then, the inverse Laplace transform algorithm of Stehfest [38] is applied to obtain the final time-dependent solutions. It is worth mentioning that several quasi-static boundary value problems are required to be solved for various values of the Laplace transform parameter. Finally, several numerical examples are presented to demonstrate the validity and accuracy of the proposed method.

2. Governing equations

Some of the governing equations for viscoelastic problems of FGMs are outlined in this section. Let us consider a transient dynamic problem in a continuously heterogeneous and linear viscoelastic domain Ω bounded by a surface Γ with an outward unit normal vector with the components n_i. The elastodynamic equilibrium equation can be written as

$$\sigma_{ij,j}(\mathbf{x},t)-\rho(\mathbf{x})\ddot{u}_i(\mathbf{x},t)+b_i(\mathbf{x},t)=0 \tag{1}$$

where $\sigma_{ij}(\mathbf{x},t)$ is the stress tensor, $b_i(\mathbf{x},t)$is the body force vector, $\rho(\mathbf{x})$ is the mass density and $u_i(\mathbf{x},t)$ is the displacement vector. It should be mentioned here that a comma denotes partial derivatives with respect to spatial variables and the conventional Einstein's summation rule over repeated indices is applied throughout the analysis. All material parameters could be dependent on spatial variables. The quasi-static problems can be considered formally as a special case of the general transient problems by omitting the acceleration term $\ddot{u}_i(\mathbf{x},t)$in the equilibrium eq. (1). Consequently, both cases are analyzed simultaneously in this article. The kinematic relations for the strain field ε_{ij} is defined by

$$\varepsilon_{ij}=\frac{1}{2}(u_{i,j}+u_{j,i}) \tag{2}$$

and the linear viscoelastic constitutive law

$$s_{ij}(\mathbf{x},t)=2\int_0^t \mu(\mathbf{x},t-\tau)\frac{\mathrm{d}e_{ij}}{\mathrm{d}\tau}\,\mathrm{d}\tau \tag{3a}$$

$$\sigma_{kk}(\mathbf{x},t)=3\int_0^t K(\mathbf{x},t-\tau)\frac{\mathrm{d}\varepsilon_{kk}}{\mathrm{d}\tau}\,\mathrm{d}\tau \tag{3b}$$

with

$$s_{ij}=\sigma_{ij}-\frac{1}{3}\sigma_{kk}\delta_{ij},\, e_{ij}=\varepsilon_{ij}-\frac{1}{3}\varepsilon_{kk}\delta_{ij} \tag{4}$$

where $s_{ij}$and e_{ij} are deviatoric components of the stress and strain tensors, respectively, δ_{ij} is the Kronecker delta, and $\mu(\mathbf{x},t)$ and $K(\mathbf{x},t)$ are the relaxation functions in shear and dilatation, respectively. Note that the relaxation functions $\mu(\mathbf{x},t)$ and $K(\mathbf{x},t)$ depend on spatial coordinates, whereas in homogeneous viscoelasticity they are only functions of time, that is, $\mu=\mu(t)$ and $K=K(t)$[10].

The corresponding boundary and initial conditions are given as follows

$$u_i(\mathbf{x},t)=\tilde{u}_i(\mathbf{x},t),\ \text{on the essential boundary } \Gamma_u \tag{5a}$$

$$t_i(\mathbf{x},t)=\sigma_{ij}(\mathbf{x},t)n_j(\mathbf{x})=\tilde{t}_i(\mathbf{x},t),\ \text{on the natural boundary } \Gamma_t \tag{5b}$$

$$u_i(\mathbf{x},t)|_{t=0}=u_i(\mathbf{x},0)\ \text{ and } \dot{u}_i(\mathbf{x},t)|_{t=0}=\dot{u}_i(\mathbf{x},0)\ \text{ in } \Omega \tag{5c}$$

where Γ_u is the part of the global boundary with prescribed displacements, while on Γ_t the traction vector is prescribed.

The correspondence principle is probably the most useful tool in viscoelasticity because the Laplace transform of the viscoelastic solution can be directly obtained from the existing elastic solution. Unfortunately, in general, the correspondence principle does not hold for FGMs. To avoid this problem, for a class of FGMs with the following separable relaxation functions

$$\mu(\mathbf{x},t)=\mu_0\tilde{\mu}(\mathbf{x})f(t) \tag{6a}$$

$$K(\mathbf{x},t)=K_0\tilde{K}(\mathbf{x})g(t) \tag{6b}$$

where μ_0 and K_0 are material constants, and $\tilde{\mu}(\mathbf{x})$,$\tilde{K}(\mathbf{x})$,$f(t)$ and $g(t)$ are nondimensional functions. Paulino and Jin [11] have shown that the correspondence principle still holds. For simplicity, the only restrictive requirement employed in the present work is the separation of the spatial and temporal variables in the relaxation functions. Substituting Eqs. (6a)-(6b) into Eqs. (3a)-(3b) results in

$$s_{ij}(\mathbf{x},t)=2\mu_0\tilde{\mu}(\mathbf{x})\int_0^t f(t-\tau)\frac{\mathrm{d}e_{ij}}{\mathrm{d}\tau}\,\mathrm{d}\tau \tag{7a}$$

$$\sigma_{kk}(\mathbf{x},t)=3K_0\tilde{K}(\mathbf{x})\int_0^t g(t-\tau)\frac{\mathrm{d}\varepsilon_{kk}}{\mathrm{d}\tau}\,\mathrm{d}\tau \tag{7b}$$

Applying the Laplace transform to Eqs. (1), (2), (7a), (7b) and boundary conditions (4a)-(4b), one obtains

$$\bar{\sigma}_{ij,j}(\mathbf{x},p)-\rho(\mathbf{x})p^2\bar{u}_i(\mathbf{x},p)+\bar{F}_i(\mathbf{x},p)=0 \tag{8a}$$

$$\bar{\varepsilon}_{ij}(\mathbf{x},p)=\frac{1}{2}(\bar{u}_{i,j}(\mathbf{x},p)+\bar{u}_{j,i}(\mathbf{x},p)) \tag{8b}$$

$$\bar{s}_{ij}(\mathbf{x},p)=2\bar{\mu}\bar{e}_{ij}(\mathbf{x},p)=2\mu_0\tilde{\mu}(\mathbf{x})p\bar{f}(p)\bar{e}_{ij}(\mathbf{x},p) \tag{8c}$$

$$\bar{\sigma}_{kk}(\mathbf{x},p)=3\bar{K}\bar{\varepsilon}_{kk}(\mathbf{x},p)=3K_0\tilde{K}(\mathbf{x})p\bar{g}(p)\bar{\varepsilon}_{kk}(\mathbf{x},p) \tag{8d}$$

$$\bar{u}_i(\mathbf{x},p)=\bar{\tilde{u}}_i(\mathbf{x},p)\quad \text{on } \Gamma_u \tag{8e}$$

$$\bar{t}_i(\mathbf{x},p)=\bar{\tilde{t}}_i(\mathbf{x},p)\quad \text{on } \Gamma_t \tag{8f}$$

where a bar over a variable means the Laplace transform, p is the Laplace transform parameter and

$$\bar{F}_i(\mathbf{x},p)=\bar{b}_i(\mathbf{x},p)+\rho(\mathbf{x})pu_i(\mathbf{x},0)+\rho(\mathbf{x})\dot{u}_i(\mathbf{x},0) \tag{9}$$

is the redefined body force in the Laplace domain. Notice that the set of Eqs. (8a)-(8f) has identical form as the Laplace transformed elastodynamic equations of FGMs with the time-independent shear modulus $\mu=\mu_0\tilde{\mu}(\mathbf{x})$ and the bulk modulus $K=K_0\tilde{K}(\mathbf{x})$, provided that the transformed viscoelastic quantities are associated with the corresponding transformed elastodynamic quantities and $\mu_0p\bar{f}(p)$ and $K_0p\bar{g}(p)$ are associated with μ_0 and K_0, respectively [15].

3. Implementation of the MLNNI method

3.1. Brief description of the natural neighbour interpolation

This section gives a brief summary of the natural neighbour interpolation (NNI), of which excellent illustrations can be referred to Sukumar et al. [32]. The NNI is one of many possible schemes for an interpolation of discrete data with reasonable accuracy, which is