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An improved numerical manifold method and its application

Y.L. Chen, L.X. Li*

State Key Laboratory for Strength and Vibration of Mechanical Structures, School of Aerospace, Xi'an Jiaotong University, Xi'an 710049, PR China

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ABSTRACT

The conventional numerical manifold method is improved in generating manifold elements. In this framework, the concept of mathematical element is underlying, and manifold elements are generated by intersecting mathematical elements with physical features. On one hand, each mathematical cover is recovered by the mathematical elements with the same node (and hence the node is the star of mathematical cover). On the other hand, each physical cover is eventually recovered by piecewise pasting the manifold elements within the same mathematical cover with the same physical feature. After this improvement, generation of manifold elements can bypass the overlapping procedure of irregular and complicated physical covers, leading to the effect as simple as the finite element method. Compared with the conventional numerical manifold method, this improvement can attain "what you see is what you get" in generating manifold elements, and construction of weight functions is theoretically straightforward. In addition, local mesh refinement becomes more convenient when necessary. Two examples are conducted to validate the improved numerical manifold method.

1. Introduction

The numerical manifold method (NMM), which was originally proposed by Shi [1,2] and even referred to as the finite cover method (FCM) [3–5], is a combination of the discontinuous deformation analysis (DDA) [6,7] and the finite element method (FEM).

In the conventional NMM, the mathematical mesh is composed of mathematical covers, and then by intersecting mathematical covers and physical features, physical covers are formed. Finally, manifold elements are obtained by overlapping physical covers. Along with it, weight functions are constructed over each mathematical cover while cover functions are chosen for each physical cover. Eventually, at the interpolation approximation, the NMM pastes two kinds of functions together on each manifold element. Therefore, the NMM retains the advantageous properties such as symmetry and high sparsity of system matrices as in the FEM. In this context, the term "physical feature" is used to include the domain in which a physical problem is defined, boundaries on which the conditions are prescribed, and interfaces between different materials, cracks, or joints.

It is no doubt that, as a numerical method, the FEM is the most widely used and hence acceptable either in scientific researches or in engineering applications. However, the FEM seems awkward in generating a mesh if complex geometries and/or physical features are present. To this end, the generalized finite element method (GFEM) [8–10] and the extended finite element method (XFEM) [11–14] were proposed to overcome this inconvenience. Nevertheless, due to the difficulty in obtaining jump function for several branches, some troubles still exist in the two methods for crack propagation problems [15]. In contrast, it is very natural for the NMM in modeling such complex discontinuities.

Since the advent, the NMM has attracted a lot of attention. Tsay et al. [16] studied the prediction of crack growth by using the NMM, and proposed a local mesh refinement and auto-remeshing scheme. Combining the virtual crack extension method, Chiou et al. [17] studied the mixed mode fracture propagation with the NMM. Terada et al. [3] used the FCM, an alias of the NMM, to study non-linear mechanical behavior, and demonstrated the equivalence to the FEM in approximation, and thereafter they first assessed the performance of the FCM [4], and then simulated progressive failure with cohesive zone fracture in heterogeneous solids and structures [5].

In recent several years, the NMM was further developed. Wong et al. [18] applied the NMM to model progressive failure in rock slopes. Zheng et al. [19] proposed a new strategy to treat crack propagation in the NMM. Zheng et al. [20] and Ding et al. [21] extended the NMM to tackling plate problems. An et al. [22] suggested the concept of weak-discontinuous physical cover in the NMM to tackle material discontinuity problem, and Yang et al. [23] proposed an approach to conduct local refinement to a triangular mesh in the NMM.

As we know, finite elements present an ease of understanding for researchers and engineers even with a little knowledge of the FEM, which is one of the major reasons why the FEM is

* Corresponding author.
E-mail address: luxianli@mail.xjtu.edu.cn (L.X. Li).

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overwhelmingly prevailing. In contrast, in the conventional NMM, manifold elements must be related to physical covers that often involve very irregular and complex geometries even for a problem with relatively simple physical features.

In order to overcome this defect, an improved NMM was proposed in this paper to generate manifold elements by intersecting mathematical elements and physical features. This method can bypass the complicated overlapping procedure of physical covers in forming manifold elements, and hence attain "what you see is what you get" as in the FEM. In Section 2, some concepts in the NMM are emphasized. The fundamentals of the improved NMM are introduced in Section 3. Several typical examples are then given in Section 4 to validate the improved NMM, and a local refinement approach is suggested when necessary. The concluding remarks are finally made in Section 5.

2. Basic concepts in the NMM

The NMM, originally proposed by Shi, provides a unified framework for both continuous and discontinuous problems. To begin with, five basic concepts of NMM are highlighted by introducing the conventional NMM.

In the conventional NMM, a mathematical mesh, i.e., the union of mathematical covers, is first given to model the problem. Mathematical cover is the first basic concept and has three properties: (1) mathematical covers are arbitrarily defined by users; (2) they are independent of, but their union must completely cover all physical features; and (3) they may overlap each other.

The second basic concept in the NMM is physical cover, which is the intersection of a mathematical cover with certain physical features. As a matter of fact, physical covers can also be understood as the subdivision of mathematical covers by physical features.

The third basic concept is manifold element, which is defined as the common regions of physical covers. As will be seen, because the numerical operations will be carried out over manifold elements, this concept is most important.

To construct the interpolation approximation, weight function $w_i(\mathbf{x})$ should be defined on mathematical cover M_i. The well-defined weight functions are required to satisfy

$$w_i(\mathbf{x}) \in C^0, \quad \mathbf{x} \in M_i$$
$$w_i(\mathbf{x}) = 0, \qquad \mathbf{x} \notin M_i \tag{1a}$$

with

$$\sum_{i=1}^{n} w_i(\mathbf{x}) = 1, \quad \mathbf{x} \in \mathbf{M}_i \tag{1b}$$

Eq. (1a) indicates that the weight function $w_i(\mathbf{x})$ is continuous over mathematical cover M_i, and non-zero only on M_i, but zero elsewhere, whereas Eq. (1b) is the partition of unity property.

In the NMM, degrees-of-freedom (DOFs) are attached to physical covers on which cover functions must be chosen. Cover function $u_j(\mathbf{x})$ on physical cover P_j can be a constant, linear or higher order of polynomial, sometimes containing other kinds of functions when necessary [15].

Hence, on manifold element e_n^m (the notation shall be explained in Section 3.2), the global approximation $\mathbf{u}(\mathbf{x})$ is eventually expressed as follows:

$$\mathbf{u}^h(\mathbf{x}) = \sum_{i,j} w_i(\mathbf{x}) u_j(\mathbf{x}) \text{ if } \mathbf{x} \in e_n^m \subseteq P_j \subseteq M_i \tag{2}$$

By virtue of Eq. (2), in the NMM, the fourth and fifth basic concepts are weight function and cover function, respectively.

3. Fundamentals of the improved NMM

In Section 2, five basic concepts of the NMM were introduced. In this section, fundamentals of the improved NMM will be presented.

3.1. Mathematical elements and mathematical covers

As mentioned in Section 2, in the conventional NMM, a mathematical mesh is the union of mathematical covers. In the improved NMM, the mesh is recognized as the union of mathematical elements, based on which mathematical covers are recovered.

3.1.1. Definition of mathematical elements

Mathematical elements are completely user-defined, and may have any shape, though, as will be seen, regular and uniform mathematical elements are recommended in practical application for the purpose of convenience and accuracy [24].

Similar to the FEM, each mathematical element is linked by nodes in order. If E_i and N_k denote mathematical elements and nodes, respectively, the connectivity can be expressed as follows:

$$E_{i,j} = N_k \tag{3}$$

For example, the mathematical mesh in Fig. 1a is regarded as the composition of two mathematical elements E_1 and E_2 shown in Fig. 1b. The two elements are linked by three or four nodes, and their connectivity is summarized in Table 1. It is noted that the connectivity of mathematical elements can be obtained easily for a regular mathematical mesh, or in aid of the matured technique in the FEM for a somewhat complex mathematical mesh.

3.1.2. Recovery of mathematical covers

Mathematical cover is an important concept in the NMM, and underlying in the conventional NMM.

In the improved NMM, each mathematical cover is however recovered by mathematical elements (already defined in Section 3.1.1) with the same node. In the NMM, the same node is termed

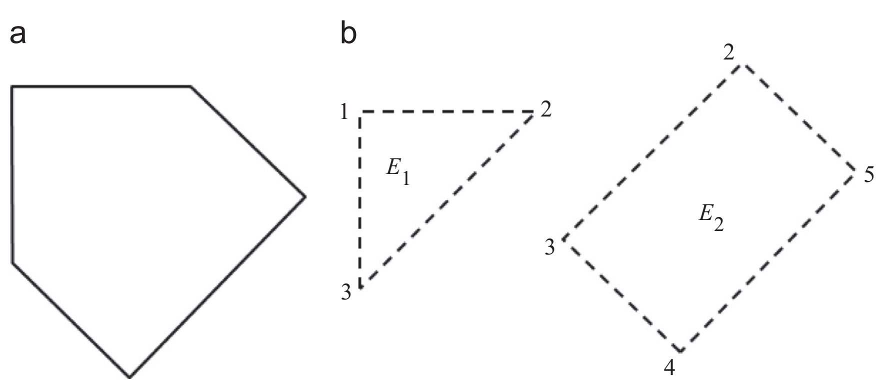

Fig. 1. A mathematical mesh composed of two mathematical elements. (a) Mathematical mesh. (b) Mathematical elements and nodes.