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The numerical manifold method for elastic wave propagation in rock with time-dependent absorbing boundary conditions

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A B S T R A C T

In this study, a modified first-order Higdon's absorbing boundary scheme is proposed and incorporated into the numerical manifold method (NMM) to reduce reflections from artificial boundaries induced by truncating infinite media. The modified time-dependent absorbing boundary scheme can not only consider the absorbing boundary and input boundary at the same artificial boundary, but also take the effects of the incident angles into consideration by adjusting the velocities and strains of points at the boundary automatically. For illustrating the efficiency of the proposed time-dependent absorbing boundary scheme, comparisons between the results of the proposed method and the widely used viscous boundary conditions for different incident angles are presented. The developed NMM is then used to investigate wave attenuation and transmission across a joint in an infinitely long rock bar.

1. Introduction

To restrict the computational domain from the infinite domain to a finite one with artificial boundaries is widely used for numerically analyzing wave propagation in an infinite media. However, the reflections introduced at the artificial boundaries may cause the wave oscillation and numerical results distortion. Therefore, additional techniques are needed to suppress non-physical reflections on the artificial boundaries.

Kausel and Peek [1] applied the Boundary Integral Method (BIM) to an infinite heterogeneous stratified soil corresponding to dynamic loads with a high accuracy. However, the BIM initially needs to seek a weak solution to the Green function which sometimes is unknown. Infinite element, which is easy to be incorporated into Finite Element Method (FEM) or Finite Difference Method (FDM), was introduced for wave propagation in infinite media by Chow and Smith [2] and Burnett [3]. However, the asymmetrical local matrix introduced by the infinite element may cause the loss of the symmetry after the near field wave discretization. The perfectly marched layer technique, which can gradually reduce the amplitude and speed of the wave, was introduced for the absorption of time-dependent wave by Berenger [4] and Zhang and Ballmann [5]. The wave speed and wave direction, which are spatial-, time- and boundary-dependent, should be firstly estimated by this technique. However, it is difficult to accurately estimate such a complicated problem. Viscous boundary condition and viscous elastic boundary condition, which replace the far field with viscous damping, were used by Lysmer and Kuhlemeyer [6] and Deeks and Randolph [7] to model the radiation of waves from the finite element mesh into the far field. Though the viscous damping is simple and easy to be achieved by numerical methods, its absorbing effect is low for some incident angles [42].

Using the first-order one-way wave equation, Higdon [8–10] constructed absorbing boundary conditions for the multi-dimensional wave equation. By adjusting the parameters in the boundary conditions, P- and S-waves induced by arbitrary incident angle can be perfectly absorbed at the numerical boundaries. By incorporating the Higdon's boundary operator, the FDM and FEM can accurately evaluate the spatial- and time-dependent wave speed and wave direction and were successfully used to solve the time domain infinite problems [11–14].

Although the Higdon's boundary operator is compatible to both continuous- and discontinuous-based numerical schemes, in all of the previous references, the numerical schemes for solving time domain infinite problems are the continuum-based methods (FDM and FEM). However, rock mass is an inhomogeneous and anisotropic geological material consisting of both continuous rock medium and discontinuous components, such as joints, cleavages, beddings and even faults. During the wave propagation, opening, closing, sliding, detaching and even evolution along these existing discontinuities can occur. Although both continuum-based and discontinnum-based methods provide useful means to analyze the wave propagation in rocks, to model such complex processes are still challenging [15–18]. Actually, the rock mass is neither continuous nor discontinuous but

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the integration of two, which can be better represented by the hybrid methods.

The Numerical Manifold Method (NMM), which is based on the partition of unity method (PUM) [19], is such a hybrid method. It combines the widely used continuum-based method FEM and discontinuous-based method discontinuous deformation analysis (DDA) in a uniform framework. The most innovative feature of the NMM is its adoption of two cover systems. By simply cutting the mathematical cover (MC) with the discontinuity, the physical cover (PC) will be separated and the discontinuity will be captured without further requirement of incorporating enrichment functions. Due to its capability in dealing with continuous–discontinuous problems, the NMM has been successfully extended to simulate the cracking involved failure problems [36–39] as well as wave propagation problems in rocks [15,20]. In previous work, either the extended computational domain or just the viscous boundary condition [6,21] was adopted. The absorbing efficiency of the incorporated scheme, however, has not been fully discussed yet.

In this study, a modified first-order Higdon's absorbing boundary scheme is incorporated into the NMM to reduce reflections from artificial boundaries induced by truncating infinite media. The modified time-dependent absorbing boundary scheme can consider both of the absorbing boundary and input boundary at the same artificial boundary. Moreover, it can also take the effects of the incident angles into consideration by adjusting the velocities and strains of points at the boundary automatically. For illustrating the efficiency of the proposed time-dependent absorbing boundary scheme, the comparisons between the results of the proposed method and the widely used viscous boundary conditions [6,21] for different incident angles are presented. The developed NMM is then used to investigate the wave attenuation and propagation across a joint in an infinitely long rock bar, which demonstrates the potential application of the developed NMM in modeling wave propagation through fractured rock at infinite media.

2. NMM for time-dependent absorbing boundary conditions

2.1. Brief introduction of the NMM

The background of the NMM was thoroughly described in the previous literatures [15,20,36–41]. As such, only the essential fundamentals are covered below.

The core and most innovative feature of the NMM is the adoption of a two cover (mesh) system, on which the nodes and elements are generated. To build a NMM model, the finite covering of a problem domain is the basic procedure.

The finite cover systems employed in the NMM are referred to the mathematical cover (MC) and physical cover (PC), respectively [22]. The MC, which is used for building PCs, can be either a mesh of regular pattern or a combination of some arbitrary figures. However, the whole mesh has to be large enough to cover the whole physical domain. The physical mesh, which includes the boundary of the material, joints, cracks, blocks and interfaces of material zones, is a unique portrait of the physical domain of a problem, and defines the integration fields. The intersection of the MC and the physical mesh, or the common area of the two systems, defines the region of the PCs. A common area of these overlapped PCs or an independent PC corresponds to an element in the NMM.

Fig. 1 illustrates the basic constructing procedures of the finite covering system adopted in the NMM. As illustrated in the figure, the mathematical meshes are formed firstly from the MCs, such as the rectangular MC M_1 and the circle MC M_2 constitute the mathematical mesh of the problem. From the formed

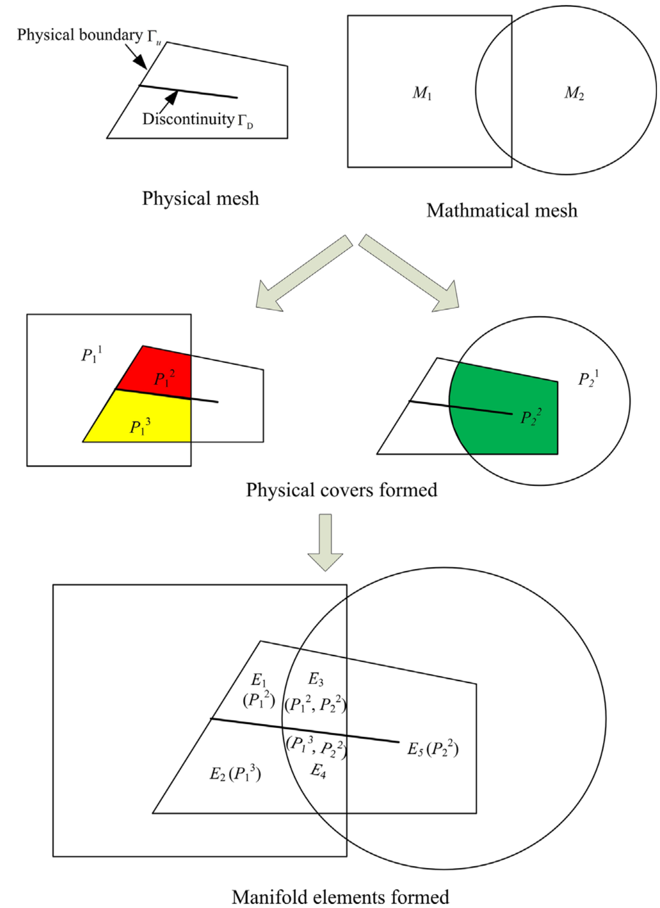

Fig. 1. Illustration of the finite cover system in NMM.

mathematical mesh and the physical boundaries, the PCs are defined. For example, MC M_1 intersecting with the physical boundary Γ_u and the discontinuity boundary Γ_D forms the PCs P_1^1, P_1^2 and P_1^3, while the MC M_2 intersecting with the physical boundary Γ_u forms PCs P_2^1 and P_2^2. Finally, the NMM elements are created by overlapping these PCs, such as the element E_3, E_4 forms from the overlapping of PCs P_1^2 and P_2^2, P_1^3 and P_2^2, respectively. The left independent areas of PCs then form the other manifold elements, such as element E_1 from PC P_1^2, E_2 from PC P_1^3 and E_5 from PC P_2^2.

On each PC P_i, a local approximation function $u_i(x)$ is independently defined. A convenient way for constructing a basis of local approximation spaces is by using the polynomial functions. e.g.

$$I^c = \{1, x, y, \ldots, x^p, x^{p-1}y, \ldots, xy^{p-1}, y^p\} \tag{1}$$

where the superscript "c" stands for conventional PCs for a two-dimensional problem.

Though the polynomials can approximate smooth functions well and capture the discontinuity directly for the conventional PCs, for PCs that are not fully intersected by the discontinuities (such as PC P_2^2 in Fig. 1), the smooth basis polynomial local approximations can neither capture the high gradient at the crack tips nor capture the jumps across the discontinuity surfaces. Therefore, special singular functions may need to be used to enrich the approximation space for capturing the singularities without refining the meshes. In this study, the linear elastic asymptotic crack-tip fields as proposed by Belyschko and Black [23], which has been already successfully incorporated into NMM for capturing the discontinuity among the singular cover [43,44], are used as suitable enrichment functions for the singular PCs. Then the enriched local approximation functions for singular PCs