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The method of fundamental solutions for complex electrical impedance tomography

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ABSTRACT

The forward problem for complex electrical impedance tomography (EIT) is solved by means of a meshless method, namely the method of fundamental solutions (MFS). The MFS for the complex EIT direct problem is numerically implemented, and its efficiency and accuracy as well as the numerical convergence of the MFS solution are analysed when assuming the presence in the medium (i.e. background) of one or two inclusions with the physical properties different from those corresponding to the background. Four numerical examples with inclusion(s) of various convex and non-convex smooth shapes (e.g. circular, elliptic, peanut-shaped and acorn-shaped) and sizes are presented and thoroughly investigated.

1. Introduction

Electrical impedance tomography (EIT) is a technique used for determining the admittivity distribution in the interior of an object, given simultaneous measurements of alternating electric currents (of frequencies varying from 10 Hz to 500 kHz) and of induced voltages on the boundary of the object [12]. For a given conductive object, the admittivity is a complex valued function whose real part is the electrical conductivity and whose imaginary part is the product of the frequency of the applied electric alternating current and the permittivity of the object. Since different materials display different electrical properties, a map of internal admittivity can be used to infer the internal structure of the object under consideration. Therefore, EIT can be used as a non-invasive and portable method of industrial, geophysical and medical imaging [1].

The reconstruction procedures proposed for static EIT include a wide range of iterative methods based on formulating the inverse problem in the framework of nonlinear optimization [2,24,40,41]. These approaches usually involve estimating the admittivity distribution of the object under consideration and then solve the forward problem (often using finite element methods [FEMs]) for the same input current patterns to compute the boundary voltages and then comparing the boundary data predicted by this estimate with the measured data. The discrepancy between these two data sets is then used to update the admittivity estimate and the procedure is repeated until a satisfactory agreement is achieved. However, the solution of the inverse problem in static EIT has suffered not only from its ill-posedness due to the inherent insensitivity of boundary measurements to any small changes of interior conductivity and permittivity values and its poor spatial resolution, but also from its reliance on accurate forward models which mimic every aspect of the imaging object (e.g. knowledge of boundary geometry, electrode positions and other sources of systematic artifacts in measured data). Hence, EIT has had limited applicability so far in clinical applications. This has encouraged the search of new reconstruction methods, such as the time-difference EIT (tdEIT) or frequency-difference EIT (fdEIT). Even though numerous tdEIT methods have been applied to image lung functions, stomach emptying or brain functions [12,31,32], there are medical applications where these time-reference data are not available (e.g. breast cancer or cerebral stroke detection). Since complex conductivity spectra of biological tissues show frequency-dependent changes [8,32], fdEIT methods have been proposed to image the changes in the admittivity distribution with respect to frequency [11,13,22,38]. It has been showed that, although there are no visible differences in the reconstructed frequency-difference images even though the true admittivity distributions have a strong frequency dependence, any anomaly can be clearly identified as long as its admittivity differs significantly from that of the background which is the case for tumour and stroke imaging.

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To achieve clinical acceptance, the theoretical developments of EIT reconstruction methods need to be closely connected with laboratory experiments and studies on real data. This implies not only the modelling of the real geometry and data collection devices but also the development of a computer software fast enough to be used for real-time monitoring. However, optimized image reconstruction techniques for EIT rely on computationally efficient and numerically robust forward solvers. In this paper, we address this need by presenting an algorithm based on the method of fundamental solutions (MFS) which can be successfully used to find the numerical solution of the EIT forward problem for piecewise constant admittivity distributions to a high level of precision.

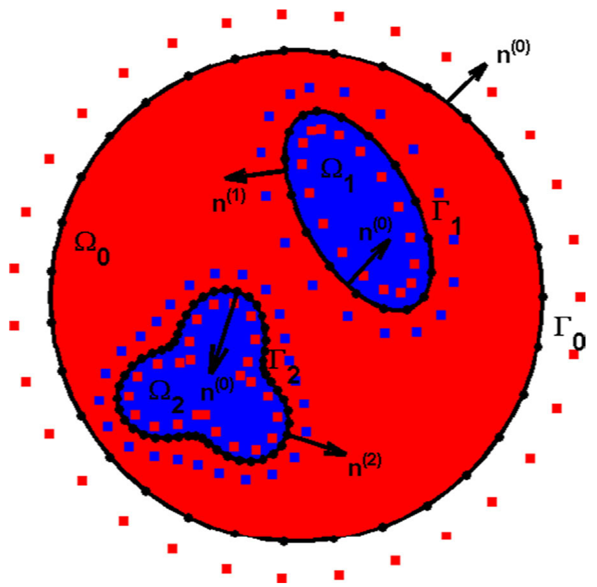

Fig. 1. Geometry of the problem. Possible placement of the sources for Ω_0 (■) and Ω_j, $j=1,2$, (■), and the collocation points (●).

The MFS is a meshless boundary collocation method applicable to boundary value problems in which a fundamental solution of the operator in the governing equation is known explicitly. The basic ideas of this method were first introduced, in the early 1960s, by Kupradze and Aleksidze [23], whilst its numerical formulation was first given by Mathon and Johnston [30] in the late 1970s. The main idea of the MFS consists of approximating the solution of the problem by a linear combination of fundamental solutions with respect to some singularities/source points which are located outside the domain. Consequently, the original problem is reduced to determining both the unknown coefficients of the fundamental solutions and the coordinates of the source points by requiring the approximation to satisfy the boundary conditions in some sense and hence solving a non-linear problem. If the source points are fixed *a priori*, then the coefficients of the MFS approximation are determined by solving a linear problem. The aforementioned MFS procedures are referred to in the literature as the *dynamic* and *static* approaches, respectively.

Despite its constraint on the knowledge of a fundamental solution of the governing partial differential equation, the MFS

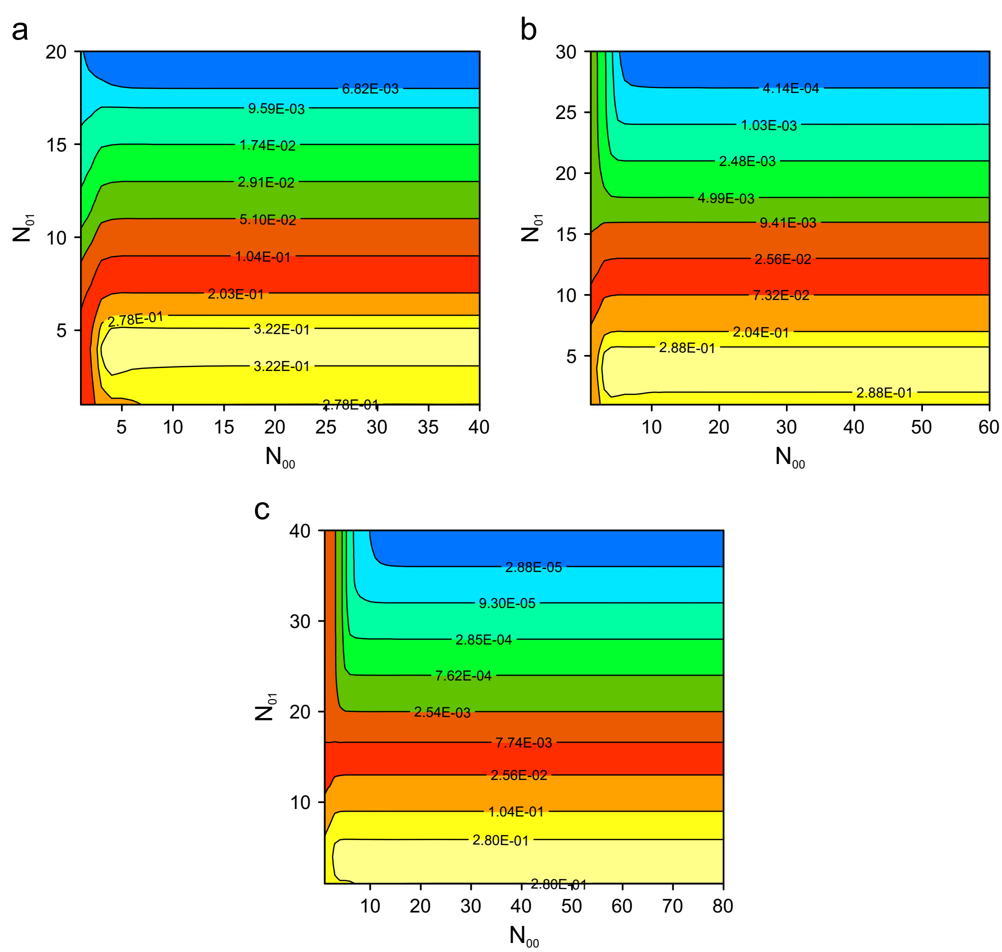

Fig. 2. The RMS error, $\mathrm{err}_{\Gamma_1}(v_0-v_1)$, as a function of the number of sources $N_{00}\in\{1,2,\dots,M_{00}\}$ and $N_{01}\in\{1,2,\dots,M_{01}\}$ at $t=0$ s and various numbers of collocation points, namely (a) $M=80$, (b) $M=120$ and (c) $M=160$, for Example 1.