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Regional connectivity in modified finite point method

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ABSTRACT

In this paper, a concept of regional connectivity has been integrated into the modified finite point method (MFPM) [33] to solve problems with different physical behaviors in adjacent regions. This approach has been employed to improve accuracy in its applications to harbor resonance of the MFPM, which has searched adjacent nodes by relative distance for local collocation. By identifying regional connectivity, only closer nodes within regions of the same regional connectivity with respect to a base point can be included for correct local collocation. In coastal engineering, phenomenon of resonance of harbors with breakwaters is a crucial consideration in harbor planning and design. Numerical computations of harbor resonance induced by monochromatic water-waves are used to verify the MFPM numerical model integrated with regional connectivity approach. The whole computational domain is divided into several subdomains, based on different physical behaviors. After numbering of each subdomain, regional connectivity is provided to exclude searching the nearest nodes from inappropriate subdomains for local collocation.

Harbors of different physical geometries, with and without breakwaters have been examined when analytical solutions [18] are available. Very good agreement between numerical results and analytical solutions has demonstrated that the concept of regional connectivity has improved the performance of MFPM. Application of this regional connectivity concept will be needed in similar problems, such as a crack in a thin plate, and a cutoff of groundwater seepage.

1. Introduction

Traditionally, the finite difference (FDM) or the finite element (FEM) methods are used to solve the partial differential equation (PDE) with boundary conditions. In general, the FDM is easy to discretize the domain of interests, but encounters difficulty in fitting discretized grids precisely on the irregular boundaries. On the contrary, the FEM is more flexible to locate grids on the irregular boundaries. In order to gain this advantage of flexibility in FEM, information of grid connectivity must be provided for each mesh or element, in addition to the positions of all grids. Preparing the information of grid connectivity is quite tedious and time-consuming, particularly if done manually.

Recently, many meshless methods are applied to establish numerical models in many researches for problems in science and engineering [9,17,32,35]. In meshless or mesh-free methods, only positions of all grids are required for data input and information of grid connectivity is no longer needed. Meshless methods can be broadly divided into two categories collocation method and Galerkin method. The Galerkin method is higher than the collocation method in accuracy and stability, but numerical integration is required. Without prior information of grids connectivity it is difficult to achieve this goal. Instead, the approach of collocation at a local base point is simple and straightforward.

Although radial basis function (known as RBF) is the most popular and widely used [7,11,31,34] in meshless methods, there are quite a number of choices of basis function for collocation method. Oñate et al. [21,22] applied finite point method (FPM), by the polynomial basis function and moving least square (MLS) to calculate convective–diffusive problem and obtained good results. Ortega et al. [23,24] applied the FPM [21,22] for shallow water equation and adopted an upwind-biased discretization for dealing with convective terms. Wang [30] studied ship waves by the finite element method (FEM), and simultaneously applied local polynomial and moving least squares approximation. One advantage of these approaches with polynomial approximations, is the gradient of the velocity potential and other physical quantities can be accurately evaluated at the same time as the approximated solutions of velocity potential are obtained. The modified finite method (MFPM) [32,33] modifies the finite point method applied by Oñate et al. [21] by using a polynomial local collocation method integrated with a moving least square (MLS) to satisfy both governing equation and boundary conditions at each nodes

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everywhere within the domain and also along the boundaries. An approach, like penalty method used in FEM, was integrated for the boundary conditions additionally. It has been shown [32,33] that better numerical approximation both within the interior domain and on the boundaries simultaneously have been obtained.

Although MFPM like other meshless methods gains the advantage of avoiding information of grid connectivity within a mesh or element as one of the input data for numerical computations, there are problems in engineering and science consisting of different regions divided by a very thin barrier or crack and behave with different physical characters, such as a breakwater in water-wave diffraction, a crack in a thin plate and a cutoff in groundwater seepage. In MFPM [32,33], the local collocation is performed by searching some nearest nodes close to a chosen base point. When physical behaviors in separated regions divided by a thin barrier, it requires to identify the nodes searched by relative distance from a base point are not from an inappropriate regions with different physical behaviors. Therefore, it requires division of the whole domain into subdomains of different physical behaviors. After numbering each subdomain, information of regional connectivity among subdomains is needed for node searching from appropriate subdomains for correct local collocation. Therefore, this approach required information of regional connectivity to identify appropriate nodes for local collocation approximation.

In this paper, phenomenon of resonance for harbors of different geometry, with or without breakwaters is examined by using MFPM with regional connectivity approach. Numerical results are compared for cases when analytical solutions are available [18].

2. Formulation of MFPM

In this paper, the numerical method of MFPM developed by Wu and Tsay [33] is employed. For completeness, a brief description of MFPM is also shown here.

Taking a general problem governed by a 2-D linear-second order PDE as an example, the governing equation is expressed as

$$\mathcal{L}\{\phi\} = c_1\phi + c_2\frac{\partial\phi}{\partial x} + c_3\frac{\partial\phi}{\partial y} + c_4\frac{\partial^2\phi}{\partial x^2} + c_5\frac{\partial^2\phi}{\partial y^2} + c_6\frac{\partial^2\phi}{\partial x\partial y} = f \tag{1}$$

and the problem is subjected to the boundary conditions

$$\mathcal{B}\{\phi\} = q_1\phi + q_2\frac{\partial\phi}{\partial x} + q_3\frac{\partial\phi}{\partial y} = g_1, \cdots \vec{x} \in \Gamma_1 \tag{2}$$

$$\phi = \phi_b, \cdots \vec{x} \in \Gamma_2 \tag{3}$$

where the coefficients c_1, c_2, c_3, c_4, c_5, c_6, q_1, q_2, q_3, f and g_1 are all functions of x and y. The entire domain can be distributed with N nodes as needed. Following the concept of local polynomial approximation in finite point methods, $\phi(\vec{x})$ can be approximated in the vicinity of a specific position $\vec{x}_j$ as the base point, so that

$$\phi(\vec{x})|_{\vec{x} \approx \vec{x}_j} \approx \hat{\phi}_j(\vec{x}) = \sum_{i=1}^{m} \alpha_{ji} p_i(\vec{X}) \tag{4}$$

in which $\vec{X} = \vec{x} - \vec{x}_j$ is the relative position vector, $p_i(\vec{X})$ is the ith monomial of the polynomial, and α_{ji} are coefficients to be determined. Here, the number of monomials depends on the dimension of $\vec{x}$ and the chosen degree of the polynomial.

The choice of the degree of the polynomial used in the local approximation depends on the problem to be solved. The dimension of $\vec{x}$ also depends on what problem is considered. For a 2-D problem governed by a second order PDE, the choices of monomials are

$$\{p_i(\vec{X}), i = 1 \sim m\} = \left\{1 \quad X \quad Y \quad X^2 \quad Y^2 \quad XY \quad X^3 \quad X^2Y \quad XY^2 \quad Y^3 \quad \ldots\right\} \tag{5}$$

in which $\vec{X} = X\vec{i} + Y\vec{j}$.

To seek a local approximation in which the difference between the exact and approximated values at $\vec{x}_j$ is smaller than those at other points around $\vec{x}_j$, one can use the function values at n points in the neighborhood of $\vec{x}_j$ and couple with the moving least squares (MLS) approach:

$$E_j = \Sigma_{k=1}^{n}\{W_{jk}[\phi(\vec{x}_k) - \hat{\phi}(\vec{x}_k)]\} \tag{6}$$

in which E_j is the weighted square error for approximating $\hat{\phi}_j(\vec{x})$ to $\phi(\vec{x})$ and W_{jk} is the weighting factor of the kth residual. The value of W_{jk} is determined by the distance between $\vec{x}_j$ and $\vec{x}_k$. Usually, it is selected using a compactly supported RBF such as the normalized Gaussian function.

$$W_{jk} = \begin{cases} \frac{\exp[-\varepsilon(r_{jk}/\rho_j)^2] - \exp(-\varepsilon)}{1-\exp(-\varepsilon)} & , \quad r_{jk} < \rho_j \\ 0 & , \quad r_{jk} \geq \rho_j \end{cases} \tag{7}$$

where r_{jk} is the distance between $\vec{x}_j$ and $\vec{x}_k$ (i.e. $r_{jk} = |\vec{x}_k - \vec{x}_j|$), ε is the shape parameter, and ρ_j is the supporting range whose subscript j denotes that it is just for the approximation in the vicinity of $\vec{x}_j$.

At each point, $\vec{x}_j$, finding a local polynomial approximation that satisfies the governing equation and the boundary conditions corresponds to minimized the value of E_j, an alternative weighted square error was introduced in Wu and Tsay [33].

$$E_j = \Sigma_{k=1}^{n}(W_{jk}(\phi(\vec{x}_k) - \hat{\phi}_j(\vec{x}_k))^2) + W'O = E_j + W'O \tag{8}$$

where W' is an additional weighting factor and

$$\mathcal{O} = (L\{\phi\} - f)^2 + (\mathcal{B}_1\{\phi\} - g_1)^2 + \ldots + (\mathcal{B}_{n_{nd}}\{\phi\} - g_{n_{nd}})^2 \tag{9}$$

where n_{nd} is the number of non-Dirichlet boundary conditions at the node $\vec{x} = \vec{x}_j$. No error-square term of Dirichlet boundary condition is included in Eq. (9) because the boundary values are already exact at these nodes. In case of $n_{nd} > 1$, it is obvious that the collocation point rests on an edge or at a corner. At internal nodes (i.e. $n_{nd} = 0$), only the first term in Eq. (9) remains. Since all the least square error terms of the solution in Eq. (8) are around the same order, choosing W' much greater than W_{jk}, whose largest value is 1, leads to $O \to 0$. The coefficients of the local polynomial can be then formulated explicitly as

$$[\alpha_j] = [\Lambda]\begin{bmatrix}\beta \\ \beta'\end{bmatrix} \tag{10}$$

in which

$$[\Lambda]_{m\times(n+n_{nd}+1)} = \left(\begin{bmatrix}A \\ A'\end{bmatrix}^T \begin{bmatrix}A \\ A'\end{bmatrix}\right)^{-1}\begin{bmatrix}A \\ A'\end{bmatrix}^T \tag{11}$$

$$[A]_{n\times m} = \begin{bmatrix} a_{11} & a_{12} & \cdots & \cdots & a_{1m} \\ a_{21} & \ddots & & & a_{2m} \\ \vdots & & a_{ki} & & \vdots \\ \vdots & & & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & \cdots & a_{nm} \end{bmatrix} \tag{12}$$

$$[A']_{(n_{nd}+1)\times m} = \begin{bmatrix} w'c_1 & \cdots & \cdots & \cdots & w'c_p & 0 & \cdots & 0 \\ w'q_{11} & \cdots & w'q_{13} & 0 & \cdots & \cdots & \cdots & 0 \\ \vdots & & \vdots & \vdots & & & & \vdots \\ w'q_{n_{nd}1} & \cdots & w'q_{n_{nd}3} & \cdots & \cdots & \cdots & \cdots & 0 \end{bmatrix} \tag{13}$$

where $w_k = \sqrt{W_{jk}}$, $\phi_k = \phi(\vec{x}_k)$, $\hat{\phi}_k = \hat{\phi}_j(\vec{x}_k)$, $a_{ki} = w_k p_i(\vec{x}_k - \vec{x}_j)$ and $w' = \sqrt{W'}$. It should be noted that this approximation is only valid in the vicinity of $\vec{x}_j$. Once a new $\vec{x}_j$ is chosen, the entries of matrix $[\Lambda]$ and components of vector $[\beta]$ and $[\beta']$ should all be renewed. In Eq. (13), the expression of zero-valued columns might be unnecessary if the degree of the local polynomial is chosen the