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Isogeometric analysis of laminated composite plates based on a four-variable refined plate theory

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ABSTRACT

In this paper, a simple and effective formulation based on isogeometric approach (IGA) and a four variable refined plate theory (RPT) is proposed to investigate the behavior of laminated composite plates. RPT model satisfies the traction-free boundary conditions at plate surfaces and describes the non-linear distribution of shear stresses without requiring shear correction factor (SCF). IGA utilizes basis functions, namely B-splines or non-uniform rational B-splines (NURBS), which reveals easily the smoothness of any arbitrary order. It hence handles easily the C^1 requirement of the RPT model. Approximating the displacement field with four degrees of freedom per each node, the present method retains the computational efficiency while ensuring the reasonable accuracy in solution.

1. Introduction

Laminated composite plates are being increasingly used in various fields of engineering such as aircrafts, aerospace, vehicles, submarine, ships, buildings, etc., because they possess many favorable mechanical properties such as high stiffness to weight and low density. Therefore a lot of research about their behaviors such as deformable characteristic, stress distribution, natural frequency and critical buckling load under various conditions haas never been stopped. Pagano [1] initially investigated the analytical three-dimensional (3D) elasticity method to predict the exact solution of simple static problems. Noor et al. [2,3] further developed 3D elasticity solution formulas for stress analysis of composite structures. It is well known that such an exact 3D approach is the most potential tool to obtain the true solution of plates. However, it is not easy to solve practical problems with complex (or even slightly complicated) geometries and boundary conditions. In addition, each layer in the 3D elasticity theory is modeled as one 3D solid and hence the computational cost of laminated composite plate analyses is increased significantly. Hence, many equivalent single layer (ESL) plate theories with suitable assumptions [4] have been then proposed to transform the 3D problem to a 2D one. Among the ESL plate theories, the classical laminate plate theory (CLPT) based on the Love–Kirchoff assumptions was first proposed. Due to ignoring the transverse shear deformation, CLPT merely provides acceptable results for the thin plate problems. The first order shear deformation theory (FSDT) based on Reissner [5] and Mindlin [6], which takes into account the shear effect, was therefore developed. In FSDT model, with the linear in-plane displacement assumption through plate thickness, the obtained shear strain/stress distributes inaccurately and does not satisfy the traction free boundary conditions at the plate surfaces. The shear correction factors (SCF) are therefore required to rectify the unrealistic shear strain energy part. The values of SCF are quite dispersed through many problems and may be difficult to determine [7]. To bypass the limitations of the FSDT, many kind of higher-order shear deformable theories (HSDT), which include higher-order terms in the displacement approximation, have then been devised such as third-order shear deformation theory (TSDT) [8–10], trigonometric shear deformation theory [11,12], exponential shear deformation theory (ESDT) [13–15], refined plate theory (RPT) and so on. The RPT model was pointed

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out by Senthilnathan et al. [16] with four unknown variables which is one variable lower than the TSDT model. Shimpi et al. [17,18] proposed RPT with just only two unknown variables using different distributed functions for the isotropic and orthotropic plates. Recently, this model is deeply researched by Thai-Huu et al. [19,20]. It is worth mentioning that the HSDT models provide better results and yield more accurate and stable solutions (e.g. inter-laminar stresses and displacements) [21,22] than the FSDT ones without requiring the SCF. However, the HSDT requires the C^1-continuity of generalized displacement field leading to the second-order derivative of the stiffness formulation. The enforcement of even C^1 continuity across inter-element boundaries in standard finite element method is not a trivial task. In the efforts to address this difficulty, several C^0 continuous elements [23–26] were then proposed or Hermite interpolation function with the C^1-continuity was taken into account in the approximation of transverse displacement [4]. Such elements may produce extra unknown variables leading to an increase in the computational cost. In this paper, we show that C^1-continuous elements will be naturally gained by using B-Spline or non-uniform rational B-Spline (NURBS) shape functions without any additional variables.

The NURBS basis functions are commonly used in the Computer Aided Design (CAD) software to describe the geometry domain [27]. They are flexible to make refinement, de-refinement, and degree elevation and gain easily the smoothness of arbitrary continuous order. Also, NURBS can be used to approximate meshfree shape functions with a desired order of consistency [28] or to merge into boundary element method to obtain the geometry and traction fields around the boundary [29]. Another way, by coupling geometry and approximations via NURBS, Hughes and co-workers have introduced a new method so-called Isogeometric Analysis (IGA) [31]. The core idea of IGA is to use same NURBS basis functions for both describing the exact geometry and constructing the finite element formulation [30]. The IGA has been well known and widely applied to various practical problems [32–39], etc.

In this paper, a formulation based on the RPT model and the isogeometric approach for static, free vibration and buckling analysis of laminated composite plates is investigated. Some higher-order distributed functions [8,13,14,17] are utilized to describe the higher-order term in the displacement field. Several numerical examples are given to show the performance of the proposed method in comparison to others in the literature.

The paper is outlined as follows. Section 2 introduces the RPT for composite plates. In Section 3, the formulation of plate theory based on IGA is described. The numerical results and discussions are provided in Section 4. Finally, this article is closed with some concluding remarks.

2. The refined plate theory

2.1. Displacement field

Regarding the effect of shear deformation, the higher-order terms are incorporated into the displacement field. A simple and famous theory for the bending plate is stated as [4]

$$\begin{aligned} u(x,y,z) &= u_0 + z\beta_x + g(z)(\beta_x + w_{,x}) \\ v(x,y,z) &= v_0 + z\beta_y + g(z)(\beta_y + w_{,y}), \quad \left(\tfrac{-h}{2} \le z \le \tfrac{h}{2}\right) \\ w(x,y) &= w_0 \end{aligned} \tag{1}$$

where $g(z) = -(4z^3/3h^2)$ and the variables $\mathbf{u}_0 = \{u_0 v_0\}^T$, w_0 and $\boldsymbol{\beta} = \{\beta_x \beta_y\}^T$ are the membrane displacements, the transverse displacement and the rotations in the *y–z*, *x–z* planes, respectively. By making additional assumptions given in Eq. (2), Senthilnathan et al. [16] proposed the refined plate theory model with one reduced variable

$$w_0 = w_b + w_s; \quad \boldsymbol{\beta} = -\nabla w_b \tag{2}$$

where w_b and w_s are defined as the bending and shear components of deflection, respectively. Eq. (1) is taken in the simpler form with four unknown variables

$$\begin{aligned} u(x,y,z) &= u_0 - z w_{b,x} + g(z) w_{s,x} \\ v(x,y,z) &= v_0 - z w_{b,y} + g(z) w_{s,y} \\ w(x,y) &= w_b + w_s \end{aligned} \tag{3}$$

The relationships between strains and displacements are described by

$$\boldsymbol{\varepsilon} = [\varepsilon_{xx}\varepsilon_{yy}\gamma_{xy}]^T = \boldsymbol{\varepsilon}_0 + z\boldsymbol{\kappa}_b + g(z)\boldsymbol{\kappa}_s \tag{4}$$

$$\boldsymbol{\gamma} = [\gamma_{xz}\gamma_{yz}]^T = f'(z)\boldsymbol{\varepsilon}_s \quad \text{in which} \quad f'(z) = g'(z) + 1 \tag{5}$$

where

$$\boldsymbol{\varepsilon}_0 = \begin{bmatrix} u_{0,x} \\ v_{0,y} \\ u_{0,y} + v_{0,x} \end{bmatrix}, \boldsymbol{\kappa}_b = -\begin{bmatrix} w_{b,xx} \\ w_{b,yy} \\ 2w_{b,xy} \end{bmatrix}, \boldsymbol{\kappa}_s = \begin{bmatrix} w_{s,xx} \\ w_{s,yy} \\ 2w_{s,xy} \end{bmatrix}, \boldsymbol{\varepsilon}_s = \begin{bmatrix} w_{s,x} \\ w_{s,y} \end{bmatrix} \tag{6}$$

From Eq. (5), an additional condition is needed to satisfy traction-free boundary condition at the top and bottom surfaces of plate. It means that $f'(z) = 0$ at $z = \pm h/2$. Based on this condition, various distributed functions $f(z)$ in forms: third-order polynomials by Reddy [8] and Shimpi [17], exponential function by Karama [13], sinusoidal function by Arya [14] and are illustrated in Table 1.

2.2. Weak form equations for plate problems

A weak form of the static model for the plates under transverse loading f_0 can be briefly expressed as

$$\int_{\Omega} \delta\boldsymbol{\varepsilon}^T \mathbf{D}^b \boldsymbol{\varepsilon} \, d\Omega + \int_{\Omega} \delta\boldsymbol{\gamma}^T \mathbf{D}^s \boldsymbol{\gamma} \, d\Omega = \int_{\Omega} \delta w f_0 \, d\Omega \tag{7}$$

where

$$\mathbf{D}^b = \begin{bmatrix} \mathbf{A} & \mathbf{B} & \mathbf{E} \\ \mathbf{B} & \mathbf{D} & \mathbf{F} \\ \mathbf{E} & \mathbf{F} & \mathbf{H} \end{bmatrix} \tag{8}$$

and the material matrices are given as

$$\begin{aligned} & A_{ij}, B_{ij}, D_{ij}, E_{ij}, F_{ij}, H_{ij} \\ & \quad = \int_{-h/2}^{h/2} (1, z, z^2, g(z), z g(z), g^2(z)) \overline{Q}_{ij} \, dz \quad (i,j = 1,2,6) \\ & D^s_{ij} = \int_{-h/2}^{h/2} [f'(z)]^2 \overline{Q}_{ij} \, dz \quad (i,j = 4,5) \end{aligned} \tag{9}$$

in which $\overline{Q}_{ij}$ are transformed material constants of the k^{th} lamina (see [4] for more detail).

Table 1
The various forms of shape function.

Model	$f(z)$	$g(z)$	$f'(z)$
Reddy [8]	$z - \frac{4}{3}z^3/h^2$	$-\frac{4}{3}z^3/h^2$	$1 - 4z^2/h^2$
Shimpi [17]	$\frac{5}{4}z - \frac{5}{3}z^3/h^2$	$\frac{1}{4}z - \frac{5}{3}z^3/h^2$	$\frac{5}{4}(1 - 4z^2/h^2)$
Karama [13]	$ze^{-2(z/h)^2}$	$ze^{-2(z/h)^2} - z$	$\left(1 - \frac{4}{h^2}z^2\right)e^{-2(z/h)^2}$
Arya [14]	$\sin\left(\frac{\pi}{h}z\right)$	$\sin\left(\frac{\pi}{h}z\right) - z$	$\frac{\pi}{h}\cos\left(\frac{\pi}{h}z\right)$