Engineering Analysis with Boundary Elements 47 (2014) 82–95

Contents lists available at ScienceDirect

Engineering Analysis with Boundary Elements

journal homepage: www.elsevier.com/locate/enganabound

Equivalent mechanical model of liquid sloshing in multi-baffled containers

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ARTICLE INFO

Article history:
Received 25 April 2013
Received in revised form
28 May 2014
Accepted 9 June 2014
Available online 17 July 2014

Keywords:
Sloshing
Equivalent mechanical model
Multi-baffled container
Boundary element method

ABSTRACT

This study presents a method to determine an equivalent mechanical model (EMM) for multi-baffled containers with arbitrary geometries. The method is implemented for 2D and axisymmetric containers. The Laplace equation and Green's theorem are used to develop the fluid model and the boundary element method (BEM) is used to solve the fluid field governing equation. Moreover, a zoning method is utilized to model arbitrary arrangements of baffles in multi-baffled containers and a reduced order model is developed to model the free-surface sloshing. The exerted hydrodynamic pressure distribution, forces and moments on the walls of the container are determined based on the Bernoulli equation and a set of recursive formulation is presented to develop the model for multi-baffled containers.

The results are validated in comparison with the literature and very good agreement is achieved. Furthermore, the effects of baffle attributes on the EMM parameters are also investigated and some conclusions are outlined.

1. Introduction

Periodic motion of fluid with free-surface in a liquid container, which is called sloshing, and its effects on the dynamic of container's supporting system is a field of interest for many researchers. The liquid sloshing leads to exert the hydrodynamic forces on the walls of the container and can cause divergence or even damage of it or failure in the system's function. So, the investigation of the hydrodynamic forces and their effects on the dynamic characteristics of the system are important concerns.

According to the importance of liquid sloshing, many valuable studies have been done in this field. Abramson [1] and Ibrahim [2] gathered lots of studies related to the sloshing in two distinct comprehensive literature. Moreover, there are some other individual research studies in this field. For example, Popov et al. [3] investigated the dynamic of liquid sloshing in compartmented and baffled rectangular road containers for some maneuvers and presented analytical steady-state and numerical transient solutions. Faltinsen and Timokha [4] developed a method to approximate the natural sloshing frequencies and modes for a 2D circular container. Wang et al. [5] determined the natural frequencies and vibration modes in a rigid cylindrical container with annular baffle.

Numerical methods are widely used in the field of fluid dynamics to analyze the liquid sloshing in the containers with complex geometries. Since the interaction of the fluid and structure occurs at their interface, it is sufficient to consider the boundary of the fluid and container for evaluating the effect of liquid sloshing on the container. That is why the BEM, which concentrates on the boundary of the fluid and structure, is an appropriate method for analyzing this kind of problems. Many researchers such as Gedikli and Erguven [6], Firouz-Abadi et al. [7], Noorian et al. [8] and Ebrahimian et al. [9] used BEM for investigation of the linear and nonlinear sloshing, sloshing frequencies and the effect of baffle on them for different container geometries.

The existence of fluid inside a system may alter its dynamic behavior. To approximate the fluid dynamics inside the containers, some EMMs have been developed. In these models, the linear planar liquid motion in a container is approximated by a series of mass–spring–dashpot systems or a set of simple pendulums. Graham and Rodriguez [10] introduced a mechanical model for liquid sloshing in a rectangular container based on linear potential theory. Roberts et al. [11] gathered design information and investigated the effects of propellant sloshing on the structural and control problems in a NASA report. Housner [12], Pinson [13], Bauer [14] and Li and Wang [15] are some other researchers who worked on the EMMs for different types of liquid containers.

Although all of the listed researchers have had significant effects on development of the EMMs for simple geometries; however, when the geometry is getting complicated, these models

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http://dx.doi.org/10.1016/j.enganabound.2014.06.001
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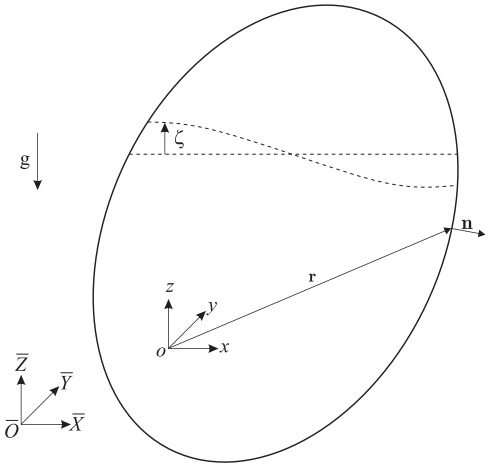

Fig. 1. Schematic view of a moving container in inertia coordinate system ($\overline{O}\,\overline{X}\,\overline{Y}\,\overline{Z}$).

lose their effectiveness. Moreover, the existence of baffle influences the dynamic behavior of the fluid and leads to change the EMM parameters which it has to be considered in the analysis.

In this study, an efficient method is presented to determine the EMM parameters of liquid sloshing in 2D and axisymmetric multi-baffled containers based on BEM formulation.

2. Governing equations

For the inviscid and irrotational flow, by assuming the existence of a function as the potential of velocity, the governing equation of the fluid can be explained by the Laplace equation [8]

$$\nabla^2\phi = 0 \tag{1}$$

where ϕ is the velocity potential function. Consider a moving coordinate system (oxyz), which is named the slosh coordinate system, with small lateral acceleration $\mathbf{a}$ and small angular velocity $\overline{\boldsymbol{\omega}}$ in the inertia coordinate system ($\overline{OXYZ}$), as shown in Fig. 1. The slosh coordinate system is so defined that its z-axis is perpendicular to the free-surface of the liquid. The non-slipping condition on the walls of a rigid container is described as follows:

$$\frac{\partial\phi}{\partial\mathbf{n}}|_{\mathrm{w}} = \mathbf{r}_n^{\mathrm{T}}\overline{\boldsymbol{\omega}} \tag{2a}$$

$$\mathbf{r}_n = (\mathbf{r}_{\mathrm{w}} \times \mathbf{n}) = [r_{nx}\ r_{ny}\ r_{nz}]^{\mathrm{T}} \tag{2b}$$

where $\mathbf{n}$ is defined as the normal outside vector of the flow field and $\mathbf{r}_{\mathrm{w}} = [x\ y\ z]^{\mathrm{T}}$ denotes the position vector of wall point in the slosh coordinate system.

Based on the definition of the velocity potential function, the kinematic condition of the free-surface and the unsteady Bernoulli equation, the following boundary condition is obtained for the free-surface of the fluid [7]:

$$\left.\frac{\partial\phi}{\partial\mathbf{n}}\right|_{\mathrm{f}} = -\frac{1}{g}\left(\mathbf{r}_{\mathrm{f}}^{\mathrm{T}}\ddot{\mathbf{a}} + \ddot{\phi}_{\mathrm{f}}\right) \tag{3}$$

where $\mathbf{r}_{\mathrm{f}}$ is the position vector of free-surface point in the slosh coordinate system and ϕ_{f} is the velocity potential function of the free-surface.

2.1. Boundary element model

Using Green's second identity and divergence theorem and assuming ϕ and $q = \partial\phi/\partial\mathbf{n}$ as well-behaved functions for a flow region with a boundary S, the following integral equation is obtained [9]:

$$c_{\mathrm{p}}\phi_{\mathrm{p}} + \int_S (\phi q^* - \phi^* q)\,\mathrm{d}S = 0 \tag{4}$$

where ϕ^* and q^* are the fundamental solution of the Laplace equation in the flow region and its derivation, respectively and c_{p} depends on the internal spatial angle at the source point p. Eq. (4) can be solved using a BEM model for 2D and axisymmetric flow fields [7–9]. By discretizing the boundary of the fluid into small elements, the following equation will be achieved [9]:

$$\mathbf{A}\boldsymbol{\phi} = \mathbf{B}\mathbf{q} \tag{5}$$

where $\mathbf{A}$ and $\mathbf{B}$ are called the influence matrices of the fluid and $\boldsymbol{\phi}$ and $\mathbf{q}$ are the vectors of nodal potential and flux density of the boundary element model, respectively.

2.2. Developing the BEM formulation for multi-baffled containers

Consider a multi-baffled 2D or an axisymmetric container as shown in Fig. 2. Based on the discretizing method which is presented in Ref. [9], one can divide a baffled container into a number of zones so that the boundary of each zone can be divided into two or three parts for interface and wall nodes, similar to Fig. 2.

For a zone with one interface, such as the first zone of the containers in Fig. 2, Eq. (5) can be written in the form of the following set of equations:

$$\begin{bmatrix} \mathbf{A}_{11}^1 & \mathbf{A}_{12}^1 \\ \mathbf{A}_{21}^1 & \mathbf{A}_{22}^1 \end{bmatrix} \begin{Bmatrix} \boldsymbol{\phi}_{\mathrm{i}}^1 \\ \boldsymbol{\phi}_{\mathrm{w}}^1 \end{Bmatrix} = \begin{bmatrix} \mathbf{B}_{11}^1 & \mathbf{B}_{12}^1 \\ \mathbf{B}_{21}^1 & \mathbf{B}_{22}^1 \end{bmatrix} \begin{Bmatrix} \mathbf{q}_{\mathrm{i}}^1 \\ \mathbf{q}_{\mathrm{w}}^1 \end{Bmatrix} \tag{6}$$

where i and w indices refer to nodes on the interface and walls, respectively. $\mathbf{A}_{kj}^1$ and $\mathbf{B}_{kj}^1$ are the associated blocks of the influence matrices of a zone with one interface. $\boldsymbol{\phi}_{\mathrm{i}}^1$, $\mathbf{q}_{\mathrm{i}}^1$, $\boldsymbol{\phi}_{\mathrm{w}}^1$ and $\mathbf{q}_{\mathrm{w}}^1$ denote the nodal potential and flux density of the first interface and the walls of the first zone, respectively. Substituting Eq. (2a) into Eq. (6) gives the following equation for the nodal potential of the walls of the first zone:

$$\boldsymbol{\phi}_{\mathrm{w}}^1 = \mathbf{D}_{\mathrm{i}}^1\boldsymbol{\phi}_{\mathrm{i}}^1 + \mathbf{D}_{\mathrm{w}}^1\overline{\boldsymbol{\omega}} \tag{7a}$$

$$\mathbf{D}_{\mathrm{i}}^1 = (\mathbf{B}_{21}^1(\mathbf{B}_{11}^1)^{-1}\mathbf{A}_{12}^1 - \mathbf{A}_{22}^1)^{-1}(\mathbf{A}_{21}^1 - \mathbf{B}_{21}^1(\mathbf{B}_{11}^1)^{-1}\mathbf{A}_{11}^1) \tag{7b}$$

$$\mathbf{D}_{\mathrm{w}}^1 = (\mathbf{B}_{21}^1(\mathbf{B}_{11}^1)^{-1}\mathbf{A}_{12}^1 - \mathbf{A}_{22}^1)^{-1}(\mathbf{B}_{21}^1(\mathbf{B}_{11}^1)^{-1}\mathbf{B}_{12}^1 - \mathbf{B}_{22}^1)\mathbf{R}_n^1 \tag{7c}$$

where $\mathbf{R}_n^1$ is a matrix that contains the $\mathbf{r}_n^{\mathrm{T}}$ of the first zone's wall points. Using these definitions, one can write the following equation for the flux density of the first interface:

$$\mathbf{q}_{\mathrm{i}}^1 = \mathbf{Z}_{\mathrm{i}}^1\boldsymbol{\phi}_{\mathrm{i}}^1 + \mathbf{Z}_{\mathrm{w}}^1\overline{\boldsymbol{\omega}} \tag{8a}$$

$$\mathbf{Z}_{\mathrm{i}}^1 = (\mathbf{B}_{11}^1)^{-1}(\mathbf{A}_{11}^1 + \mathbf{A}_{12}^1\mathbf{D}_{\mathrm{i}}^1) \tag{8b}$$

$$\mathbf{Z}_{\mathrm{w}}^1 = (\mathbf{B}_{11}^1)^{-1}(\mathbf{A}_{12}^1\mathbf{D}_{\mathrm{w}}^1 - \mathbf{B}_{12}^1\mathbf{R}_n^1) \tag{8c}$$

where $\mathbf{Z}_{\mathrm{i}}^1$ and $\mathbf{Z}_{\mathrm{w}}^1$ are the interface influence matrices of the first zone.

For the mth zone of the container which has more interfaces, Eq. (5) can be written as the following set of equations [9]:

$$\begin{bmatrix} \mathbf{A}_{11}^m & \mathbf{A}_{12}^m & \mathbf{A}_{13}^m \\ \mathbf{A}_{21}^m & \mathbf{A}_{22}^m & \mathbf{A}_{23}^m \\ \mathbf{A}_{31}^m & \mathbf{A}_{32}^m & \mathbf{A}_{33}^m \end{bmatrix} \begin{Bmatrix} \boldsymbol{\phi}_{\mathrm{i}}^m \\ \boldsymbol{\phi}_{\mathrm{w}}^m \\ \boldsymbol{\phi}_{\mathrm{i}}^{m-1} \end{Bmatrix} = \begin{bmatrix} \mathbf{B}_{11}^m & \mathbf{B}_{12}^m & \mathbf{B}_{13}^m \\ \mathbf{B}_{21}^m & \mathbf{B}_{22}^m & \mathbf{B}_{23}^m \\ \mathbf{B}_{31}^m & \mathbf{B}_{32}^m & \mathbf{B}_{33}^m \end{bmatrix} \begin{Bmatrix} \mathbf{q}_{\mathrm{i}}^m \\ \mathbf{q}_{\mathrm{w}}^m \\ -\mathbf{q}_{\mathrm{i}}^{m-1} \end{Bmatrix} \tag{9}$$

By substituting Eq. (2a) into Eq. (9), one can achieve the nodal flux density of the mth interface as follows:

$$\mathbf{q}_{\mathrm{i}}^m = \mathbf{Z}_{\mathrm{i}}^m\boldsymbol{\phi}_{\mathrm{i}}^m + \mathbf{Z}_{\mathrm{w}}^m\overline{\boldsymbol{\omega}} \tag{10a}$$