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Node adaptation for global collocation with radial basis functions using direct multisearch for multiobjective optimization

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ABSTRACT

Meshless methods are used for their capability of producing excellent solutions without requiring a mesh, avoiding mesh related problems encountered in other numerical methods, such as finite elements. However, node placement is still an open question, specially in strong form collocation meshless methods. The number of used nodes can have a big influence on matrix size and therefore produce ill-conditioned matrices. In order to optimize node position and number, a direct multisearch technique for multiobjective optimization is used to optimize node distribution in the global collocation method using radial basis functions. The optimization method is applied to the bending of isotropic simply supported plates. Using as a starting condition a uniformly distributed grid, results show that the method is capable of reducing the number of nodes in the grid without compromising the accuracy of the solution.

1. Introduction

Collocation with radial basis functions is increasingly used in order to solve systems of partial differential equations. This truly meshless method is easy to implement and is adequate to complex geometries, since no integrations in the domain are needed. In the collocation with radial basis function scheme it is assumed that any function, f may be written as a combination of N continuously differentiable basis functions, g,

$$f(\mathbf{x}) = \sum_{j=1}^{N} \beta_j g_j(\mathbf{x} - \mathbf{x}_j, \varepsilon) \tag{1}$$

where g_j depends on a distance d between N grid nodes with coordinates $\mathbf{x}$ and may depend on a shape parameter ε. The shape parameter, sometimes referred as a 'fine tuner' is a non-zero input parameter defined by the user. The user defined shape parameter is a positive constant that may cause accuracy issues [1–4]. The use of a radial basis function (the multiquadric radial basis function) for interpolation was proposed by Hardy and later considered by Franke as one of the best methods in terms of accuracy for scatter data interpolation [5,6].

Kansa's unsymmetrical collocation method was used for the solution of boundary-value problems. This method produces dense, unsymmetrical, ill-conditioned matrices. High accuracy can be obtained if adequate shape parameter is chosen.

Some authors use a simple expression with a constant shape parameter for all grid nodes that considers an evolution of the shape parameter related to the number of grid nodes [3]. Another approach proposed by Kansa consists in using different values for shape parameters at different node locations; usually a higher value is used near boundaries [1,2].

Some optimization techniques have been proposed to choose a good shape parameter. Rippa and Wang used a cross validation technique for shape parameter optimization in multiquadric interpolation [7,8]. The concept was extended by Roque and Ferreira to Kansa's method for solving systems of PDEs [9]. Using a cross validation technique it is possible to obtain good solutions for plate bending problems, even with a reduced number of grid nodes, for regular and irregular node distribution.

Another open issue that affects the quality of solutions is the distribution of nodes for interpolation. Michelli demonstrated that multiquadric surface interpolation is always solvable, for distinct data sets [10]. Although any grid may be used, experience shows that different node distributions produce different results. Therefore, a given global error can be obtained with different number of nodes and position.

In order to optimize node distribution for global collocation method with radial basis function, some proposed techniques use node adaptive grid strategies, usually using an error estimate to determine the node insertion/remotion strategy [11]. Sarra used

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node adaptive method for 1D time dependent partial differential equations [12]. Casanova et al. presented domain decomposition technique with a node adaptive algorithm to solve PDEs [13]. Hon et al. [14] and Schaback and Wendland [15] used an adaptive greedy algorithm to optimized node distribution when dealing with large radial basis functions systems. An adaptive technique was also used by Hon for solving problems with boundary layer [16].

More recently, Shanazari and Hosami used an equi-distribution strategy to adapt node position for irregular regions [17] and Esmaeilbeigi and Hosseini introduce a dynamic algorithm to perform a local node adaptive strategy in nearly singular regions [18].

Many problems in engineering depend on known discrete data, not always uniformly distributed over the problem's domain and many times scarce. Although the numerical method is easy to apply to complex geometries and irregular grids, choosing a good shape parameter and/or a good node distribution is important to produce accurate solutions, especially in the presence of sparse node grids. This is not an easy task when using irregular grids, since most studies involving collocation with radial basis functions are related to uniformly distributed grids. In this paper, the direct multisearch method is used to find optimal node grid distributions for a previously optimized shape parameter, ε. Results show that although with a higher computational cost due to optimization, the method is capable of finding good solutions for highly irregular grids. The problem of plate bending proposed in the present paper involves a system of partial differential equations with three distinct variables (u_w, ϕ_x and ϕ_y) corresponding to the plate vertical displacement and two rotations about x- and y-axes, respectively. The authors present, for the first time, a node adaptation strategy based on a multiobjective optimization technique to analyze the bending of simply supported plates using a meshless global collocation with radial basis functions. The objective is to minimize errors for all three variables u_w, ϕ_x and ϕ_y and also minimize the number of nodes in the grid, N. Since these can be conflicting objectives, the use of a multiobjective optimization technique allows to find not only one unique solution, but a set of solutions, also known as Pareto solutions. The multiobjective problem is solved using the direct multisearch (DMS) method [19].

2. Global collocation for PDE

Consider a boundary problem with domain $\Omega \in \mathbb{R}^n$ and with an elliptic differential equation given by

$$\begin{cases} Lu(x) = s(x) \ x \in \Omega \subset \mathbb{R}^n \\ Bu(x) = f(x) \ x \in \ \partial\Omega \subset \mathbb{R}^n \end{cases} \tag{2}$$

where L and B are differential operators in domain Ω and in boundary $\partial\Omega$, respectively. Nodes $(\mathbf{x}_j, j = 1, \ldots, N_B)$ and $(\mathbf{x}_j, j = N_B + 1, \ldots, N)$ are distributed in the boundary and on the domain, respectively. The solution $u(\mathbf{x})$ is approximated by $\tilde{u}$,

$$\tilde{u}(\mathbf{x}) = \sum_{j=1}^{N} \beta_j g(\| \mathbf{x} - \mathbf{x}_j \|, \varepsilon) \tag{3}$$

and inserting L and B operators in Eq. (3) we obtain the following equations:

$$\begin{cases} \tilde{u}_B(\mathbf{x}) \equiv \sum_{j=1}^{N} \beta_j Bg(\| \mathbf{x} - \mathbf{x}_j \|, \varepsilon) = f(\mathbf{x}_i), \quad i = 1, \ldots, N_B \\ \tilde{u}_L(\mathbf{x}) \equiv \sum_{j=1}^{N} \beta_j Lg(\| \mathbf{x} - \mathbf{x}_j \|, \varepsilon) = s(\mathbf{x}_i), \quad i = N_B + 1, \ldots, N \end{cases} \tag{4}$$

where $f(\mathbf{x}_i)$ and $s(\mathbf{x}_i)$ are the prescribed values on boundary nodes and domain nodes, respectively. Solving the previous system in order to β, it is possible to interpolate the solution by using Eq. (3).

In the present paper, the multiquadric radial basis function is considered,

$$g = \sqrt{1 + (\varepsilon r)^2} \tag{5}$$

where r is the Euclidian distance between distinct grid nodes and ε is a shape parameter.

3. First-order shear deformation theory

In this section, we briefly present the basic equations for the first-order shear deformation theory (FSDT) for plates. A more detailed review can be found in Reddy [20]. We seek the equations of motion and the discretization of such equilibrium equations and boundary conditions, by RBF interpolation.

Considering static analysis and isotropic plates, the displacement field for the first order shear deformation theory is

$$\begin{aligned} U(x, y, z) &= z\phi_x(x, y) \\ V(x, y, z) &= z\phi_y(x, y) \\ W(x, y, z) &= w(x, y) \end{aligned} \tag{6}$$

where U and V are the inplane displacements at any point (x, y, z), w is the transverse deflection, ϕ_x and ϕ_y are the rotations of the normals to the midplane about the y- and x-axes, respectively. The thickness of the plate is denoted as h.

The strain–displacement relationships are given as

$$\begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \\ \gamma_{xz} \\ \gamma_{yz} \end{Bmatrix} = \begin{Bmatrix} \frac{\partial U}{\partial x} \\ \frac{\partial V}{\partial y} \\ \frac{\partial U}{\partial y} + \frac{\partial V}{\partial x} \\ \frac{\partial U}{\partial z} + \frac{\partial W}{\partial x} \\ \frac{\partial V}{\partial z} + \frac{\partial W}{\partial y} \end{Bmatrix} \tag{7}$$

Therefore strains can be expressed as

$$\begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \\ \gamma_{xz} \\ \gamma_{yz} \end{Bmatrix} = \begin{Bmatrix} \varepsilon_{xx}^{(0)} \\ \varepsilon_{yy}^{(0)} \\ \gamma_{xy}^{(0)} \\ \gamma_{xz}^{(0)} \\ \gamma_{yz}^{(0)} \end{Bmatrix} + z \begin{Bmatrix} \varepsilon_{xx}^{(1)} \\ \varepsilon_{yy}^{(1)} \\ \gamma_{xy}^{(1)} \\ \gamma_{xz}^{(1)} \\ \gamma_{yz}^{(1)} \end{Bmatrix}, \tag{8}$$

where

$$\begin{Bmatrix} \varepsilon_{xx}^{(0)} \\ \varepsilon_{yy}^{(0)} \\ \gamma_{xy}^{(0)} \\ \gamma_{xz}^{(0)} \\ \gamma_{yz}^{(0)} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ \frac{\partial w}{\partial x} + \phi_x \\ \frac{\partial w}{\partial y} + \phi_y \end{Bmatrix}, \quad \begin{Bmatrix} \varepsilon_{xx}^{(1)} \\ \varepsilon_{yy}^{(1)} \\ \gamma_{xy}^{(1)} \\ \gamma_{xz}^{(1)} \\ \gamma_{yz}^{(1)} \end{Bmatrix} = \begin{Bmatrix} \frac{\partial \phi_x}{\partial x} \\ \frac{\partial \phi_y}{\partial y} \\ \frac{\partial \phi_x}{\partial y} + \frac{\partial \phi_y}{\partial x} \\ 0 \\ 0 \end{Bmatrix}. \tag{9}$$

The stress–strain relations can be expressed as

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \tau_{xy} \\ \tau_{xz} \\ \tau_{yz} \end{Bmatrix} = \begin{bmatrix} \frac{E}{1-\nu^2} & \frac{\nu E}{1-\nu^2} & 0 & 0 & 0 \\ \frac{\nu E}{1-\nu^2} & \frac{E}{1-\nu^2} & 0 & 0 & 0 \\ 0 & 0 & G & 0 & 0 \\ 0 & 0 & 0 & G & 0 \\ 0 & 0 & 0 & 0 & G \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \\ \gamma_{xz} \\ \gamma_{yz} \end{Bmatrix} \tag{10}$$

in which E, $\nu$and $G = E/2(1+\nu)$ are materials properties.

The equations of motion of the first-order theory are derived from the principle of virtual displacements [20]. The virtual strain energy (δU), and the virtual work done by applied forces (δV) are