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Null-field integral approach for the piezoelectricity problems with multiple elliptical inhomogeneities

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ABSTRACT

Based on the successful experience of solving anti-plane problems containing multiple elliptical inclusions, we extend to deal with the piezoelectricity problems containing arbitrary elliptical inhomogeneities. In order to fully capture the elliptical geometry, the keypoint of the addition theorem in terms of the elliptical coordinates is utilized to expand the fundamental solution to the degenerate kernel and boundary densities are simulated by the eigenfunction expansion. Only boundary nodes are required instead of boundary elements. Therefore, the proposed approach belongs to one kind of meshless and semi-analytical methods. Besides, the error stems from the number of truncation terms of the eigenfunction expansion and the convergence rate of exponential order is better than the linear order of the conventional boundary element method. It is worth noting that there are Jacobian terms in the degenerate kernel, boundary density and contour integral. However, they would cancel each other out in the process of the boundary contour integral. As the result, the orthogonal property of eigenfunction is preserved and the boundary integral can be easily calculated. For verifying the validity of the present method, the problem of an elliptical inhomogeneity in an infinite piezoelectric material subject to anti-plane shear and in-plane electric field is considered to compare with the analytical solution in the literature. Besides, two circular inhomogenieties can be seen as a special case to compare with the available data by approximating the major and minor axes. Finally, the problem of two elliptical inhomogeneities in an infinite piezoelectric material is also provided in this paper.

1. Introduction

In recent years, more and more investigators paid their attention to study the actuators and sensors because they were widely used in smart materials or structures technology. Therefore, the study of electromechanical behavior of piezoelectric material becomes an important issue. It is well known that it results in the stress concentration when the inhomogeneities or defects exist in the materials. In this article, we extend the previous works [1] on the piezoelectricity problems with "circular" inclusions to deal with the problem containing "elliptical" inhomogeneities.

For an elliptical shape, it may be more general than a circular geometry in the practical applications. Based on the concept of complex potential, Gong and Meguid [2] used the conformal mapping and the Laurent series expansion to solve an infinite medium containing an elliptical inhomogeneity under the anti-plane shear. Explicit form of the stress function in the inhomogeneity as well as in the matrix was derived in their work. Then, a generalized and unified treatment was developed by Gong [3] for the elliptical inclusion embedded in an infinite matrix not only under the remote shear but also interacting by the screw dislocation. Besides, Shen et al. [4] developed a semi-analytical solution for the problem of an elliptical inclusion not perfectly bonded in an infinite matrix under the anti-plane shear. Under the assumption of continuous tractions and discontinuous displacements across the interface, they used a model of a spring layer with thickness to simulate the interface. They found that the non-uniform stress field and the average stresses in the inclusion is highly related to the aspect ratio of the inclusion and the parameter of interface simulation. Recently, Chen used the complex variable boundary integral equation (CVBIE) to solve the problems containing elliptical inclusions [5–7]. In his work, he focused on the problems of plane elasticity. Chen [8] also discussed with the anti-plane problem and derived a closed-form solution by using complex variable and conformal mapping. However, the problem he solved is simpler since only one elliptical inclusion is considered. For arbitrary distributed elliptical inclusions under remote shears, few works were found in literature. To the authors' knowledge, Noda and Matsuo [9] have used the Cauchy-type singular integral equations to solve an interaction problem of elliptical inclusions distributed in an infinite

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medium under a longitudinal shear loading. They discussed different outlet of two elliptical inclusions as well as different ratios of shear moduli. Later, Lee and Kim [10] also revisited the problem of Noda and Matsuo [9] by using the volume integral equation method. Lee and Chen [11] also successfully used the null-field boundary integral equation in conjunction with degenerate kernels to solve the problem. Besides, we do not find other works to discuss on this issue containing more than two inclusions.

For the piezoelectricity problems with circular inclusions, many researchers [12–16] made much contribution on this issue. However, for containing elliptical inhomogenieties, Meguid and Zhong [17] used the complex-variable method to study the problem of a piezoelectric elliptical inhomogeneity. They derived the analytical solution in their works. Pak [18] used the conformal mapping technique to obtain a closed-form solution. The previous works were very similar. The main difference is that Meguid and Zhong [17] provided a general series solution, but Pak [18] derived an explicit closed-form solution. Besides, numerous researchers have successfully solved similar problems with an elliptical inclusion. However, to the authors' best knowledge, we do not find any work on dealing with anti-plane piezoelectric problems containing two or more than two elliptical inclusions in the literature. This is our main concern.

In this paper, we extend the successful experience of solving piezoelectricity with circular inclusions to deal with the problem containing elliptical holes and/or inclusions. The problems of arbitrary location, different orientation, various size and any number of elliptical holes and/or inhomogeneities imbedded in an isotropic and infinite medium are considered. By fully employing the elliptical geometry, fundamental solutions were expanded into the degenerate kernel by using an addition theorem in terms of the elliptical coordinates, and boundary densities are approximated by the eigenfunction expansion. The proposed approach can be seen as one kind of meshless and semi-analytical methods because only collocation points on the real boundary are required and the error purely attributes to the number of truncation terms. The case containing an elliptical inhomogeneity is used to verify the validity of the present approach. Besides, in order to show the generality and the accuracy for solving two or more than two elliptical inclusions, the available result of two circular-inclusion case is used to compare with the solution of the present approach by numerically approaching the length of the major axis to be almost equal to the minor axis. Finally, the numerical results of two real elliptical inhomogeneities in an infinite piezoelectric material are also provided by using the present approach in this paper.

2. Problem statement and formulation

2.1. Problem statement

The problem to be considered here is an infinite piezoelectric medium with multiple elliptical inclusions under the remote anti-plane shears (σ_{zx}^{∞} and σ_{zy}^{∞}) and the in-plane far-field electric field (E_x^{∞} and E_y^{∞}) as shown in Fig. 1. Bleustein [12] has pointed out that if one takes the plane normal to the poling direction as the plane of interest, only the anti-plane displacement (w) couples with the in-plane electric fields (E_x and E_y). Therefore, only the anti-plane displacement and in-plane electric field are considered in this article such as u, v and E_z are the vanishing components. In the absence of the body forces and body charges, the governing equations coupled by the displacement and electric potential can be obtained as follows:

$$c_{44}\nabla^2 w + e_{15}\nabla^2 \Phi = 0, \tag{1}$$

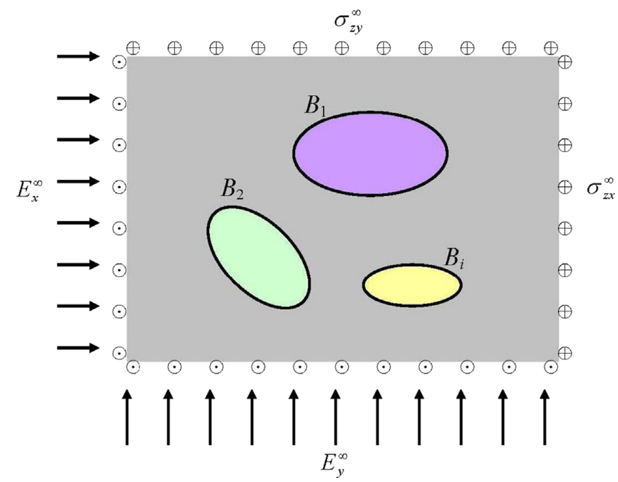

Fig. 1. Sketch of the problem.

$$e_{15}\nabla^2 w - \varepsilon_{11}\nabla^2 \Phi = 0, \tag{2}$$

where ∇^2 is the two-dimensional Laplacian operator, c_{44} is the elastic modulus, e_{15} is the piezoelectric constant, ε_{11} is the dielectric constant, w is the anti-plane displacement and Φ is the in-plane electric potential. From Eqs. (1) and (2), we can simplify the equations as

$$\nabla^2 w = 0 \quad \text{and} \quad \nabla^2 \Phi = 0 \tag{3}$$

The constitutive equations coupled between the elastic filed and electric field are

$$\sigma_{zx} = c_{44}\gamma_{zx} - e_{15}E_x, \quad \sigma_{zy} = c_{44}\gamma_{zy} - e_{15}E_y, \tag{4}$$

$$D_x = e_{15}\gamma_{zx} + \varepsilon_{11}E_x, \quad D_y = e_{15}\gamma_{zy} + \varepsilon_{11}E_y, \tag{5}$$

where γ_{zx} and γ_{zy} are the anti-plane shear strains, and D_x and D_y are the in-plane electric displacements. By employing the technique of taking free body, the problem can be decomposed into two parts. One is an infinite piezoelectric medium with N elliptical holes (Fig. 2(a)) and the other is only N individual inclusions problem (Fig. 2(b)). For the problem in Fig. 2(a), it can be superimposed by two parts as shown in Fig. 3(a) and (b). Both the two parts in Figs. 2(b) and 3(b) satisfy the Laplace equations as shown in Eq. (3). Besides, the interface between the matrix and inclusion is assumed perfectly bonded and it satisfies the following interface condition for stress fields and electric fields,

$$w^M = w^I \quad \text{and} \quad \sigma_{z\xi}^M = \sigma_{z\xi}^I \quad \text{on} \quad B_k, \tag{6}$$

$$\Phi^M = \Phi^I \quad \text{and} \quad D_\xi^M = D_\xi^I \quad \text{on} \quad B_k, \tag{7}$$

under the elliptic coordinates.

2.2. Dual null-field boundary integral formulation

2.2.1. Conventional version

The integral equation for the domain point can be derived from the third Green's identity, we have

$$w(\mathbf{x}) = \int_B T(\mathbf{s},\mathbf{x})\, w(\mathbf{s})\, dB(\mathbf{s}) - \int_B U(\mathbf{s},\mathbf{x})\, t(\mathbf{s})\, dB(\mathbf{s}), \quad \mathbf{x} \in D, \tag{8}$$

$$t(\mathbf{x}) = \int_B M(\mathbf{s},\mathbf{x})\, w(\mathbf{s})\, dB(\mathbf{s}) - \int_B L(\mathbf{s},\mathbf{x})\, t(\mathbf{s})\, dB(\mathbf{s}), \quad \mathbf{x} \in D, \tag{9}$$