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Is the Burton–Miller formulation really free of fictitious eigenfrequencies?

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ABSTRACT

This paper is concerned with the fictitious eigenfrequency problem of the boundary integral equation methods when solving exterior acoustic problems. A contour integral method is used to convert the nonlinear eigenproblems caused by the boundary element method into ordinary eigenproblems. Since both real and complex eigenvalues can be extracted by using the contour integral method, it enables us to investigate the fictitious eigenfrequency problem in a new way rather than comparing the accuracy of numerical solutions or the condition numbers of boundary element coefficient matrices. The interior and exterior acoustic fields of a sphere with both Dirichlet and Neumann boundary conditions are taken as numerical examples. The pulsating sphere example is studied and all fictitious eigenfrequencies corresponding to the related interior problem are observed. The reasons are given for the usual absence of many fictitious eigenfrequencies in the literature. Fictitious eigenfrequency phenomena of the Kirchhoff–Helmholtz boundary integral equation, its normal derivative formulation and the Burton–Miller formulation are investigated through the eigenvalue analysis. The actual effect of the Burton–Miller formulation on fictitious eigenfrequencies is revealed and the optimal choice of the coupling parameter is confirmed.

1. Introduction

The boundary element method (BEM) has emerged as a powerful numerical tool for decades to solve many engineering problems, especially acoustic and electromagnetic problems. In contrast to the finite element method (FEM), BEM has some advantages, such as the high accuracy, the reduction of dimensionality by one and the incomparable superiority in solving semi-infinite or infinite wave propagation problems [1,2]. However, some inherent shortcomings also exist [1,2]. One of them is the fictitious eigenfrequency problem, also called the non-uniqueness difficulty or the non-unique solution difficulty. It is well-known that BEM based on the Kirchhoff–Helmholtz boundary integral equation fails to yield unique solutions for exterior acoustic problems at the eigenfrequencies of the associated interior problems [3]. These eigenfrequencies are usually called fictitious eigenfrequencies because they have no physical meaning, but just arise from the drawback of the boundary integral representation when solving exterior acoustic problems. Over the last several decades, a number of methods and formulations have been proposed to tackle this problem, as surveyed in Ref. [4]. Among them, two main methods appropriate for practical applications have been widely applied. One is the combined Helmholtz integral equation formulation (CHIEF) [3] which can successfully conquer the problem at low frequencies [5]. The other is the Burton–Miller formulation [6] which has been proved to circumvent the fictitious eigenfrequency problem at all frequencies [5]. In the Burton–Miller formulation, the Kirchhoff–Helmholtz boundary integral equation and its normal derivative formulation are combined together with a properly chosen coupling parameter. So far, many applications of the Burton–Miller formulation further confirm the effectiveness of the method; cf. [4] and references therein. However, is the Burton–Miller formulation really free of fictitious eigenfrequencies? What happens to fictitious eigenfrequencies when using the Burton–Miller formulation? In order to get the answers to these questions, a boundary element eigenvalue analysis technique is implemented in this paper.

Usually, eigenproblems can be solved naturally using FEM [7–10]. However, since the coefficient matrices involve wave number implicitly, the original eigenproblem for the Helmholtz equation becomes a

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nonlinear eigenproblem when formulated by BEM. Therefore, it is not an easy task to solve acoustic eigenproblems using BEM. In order to solve such problems, a number of transform methods have been proposed [11–21], including the dual reciprocity method [11,12], the particular integral method [13,14], the multiple reciprocity method [15] and their applications [16–20]. Also, a numerical eigenvalue analysis by the Galerkin BEM has been carried out in the framework of the concept of eigenproblems for holomorphic Fredholm operator-valued functions [22]. Advantages and disadvantages for some of these methods have been discussed by Kamiya et al. [23] and Ali et al. [24]. Furthermore, nonlinear eigenproblems of the vibro-acoustic simulations formulated by coupled finite and boundary element approaches have been solved in Refs. [25–27]. In addition to these methods, the contour integral methods (CIM) have been recently developed [28,29]. In this kind of methods, a nonlinear eigenproblem is converted into a generalized eigenproblem whose dimension is much smaller than the original one. The conversion is achieved by solving a set of linear systems of equations, for instance the standard boundary element systems of equations in the BEM eigenvalue analysis. CIM has already been applied to solve nonlinear eigenproblems formulated by the acoustic BEM [30,31] and the method of fundamental solution (MFS) [32], and also nonlinear eigenproblems of vibro-acoustic simulations formulated by a coupled finite and boundary element approach [33]. In this paper, a CIM approach called the block Sakurai–Sugiura (SS) method [28] is employed. Different from other transform methods, CIM is implemented in the complex domain, consequently it can also extract eigenvalues with large imaginary parts. These complex eigenvalues cause more effort to solve the problems, but they are usually not of interest. Therefore, this is evidently a shortcoming of CIM in usual applications. However, in the following it will be shown that this is quite the reason for the choice of CIM in this paper.

In addition to the manipulation of strongly- and hyper-singular boundary integrals included in the Burton–Miller formulation, the choice of the coupling parameter in the formulation is also very important. Burton and Miller in their pioneer paper [6] proved mathematically that setting the imaginary part of the coupling parameter nonzero guarantees unique solutions for exterior Neumann problems. Meyer et al. [34] later recommended the choice of the parameter as i/k based on numerical experiments, where i and k are the imaginary unit and wave number, respectively. Terai [35] clarified that the value as i/k is optimal for a time factor of $e^{-i\omega t}$ whereas in case of $e^{i\omega t}$, the optimal value should be $-i/k$. Optimal values for the coupling parameter were proposed as functions of wave number in order to minimize the condition numbers of the appropriate integral operators in Refs. [36] and [37] for the indirect and direct BEM approaches, respectively. The validity of the Burton–Miller formulation for exterior Dirichlet problems was also confirmed in Refs. [36,37] through numerical experiments. In this paper, the optimal choice of the coupling parameter is confirmed in a new way instead of the solution accuracy analysis in Ref. [34] and minimizing condition numbers in Refs. [36,37]. Also, the validity of the Burton–Miller formulation for exterior Dirichlet problems are demonstrated through the fictitious eigenvalue analysis.

The remainder of this paper is organized as follows. BEM formulations for acoustic problems are reviewed in Section 2. Nonlinear eigenvalue analysis using the block SS method is introduced in Section 3. The interior and exterior fields of a unit sphere with Dirichlet and Neumann boundary conditions are analyzed in Section 4. Section 5 concludes the paper with further discussions.

2. BEM formulations

The Helmholtz equation which is the governing equation in steady-state linear acoustics can be reformulated into a Kirchhoff–Helmholtz boundary integral equation defined on the structural boundary Γ as follows:

$$c(\boldsymbol{x})p(\boldsymbol{x})+\int_\Gamma q^*(\boldsymbol{x},\boldsymbol{y})p(\boldsymbol{y})\,\mathrm{d}\Gamma(\boldsymbol{y})=\int_\Gamma p^*(\boldsymbol{x},\boldsymbol{y})q(\boldsymbol{y})\,\mathrm{d}\Gamma(\boldsymbol{y}), \tag{1}$$

where the coefficient $c(\boldsymbol{x})$ depends on the position of the source point $\boldsymbol{x}$ and is 1/2 when $\boldsymbol{x}$ is located on a smooth part of the boundary, $p(\boldsymbol{x})$ is the sound pressure, $p^*(\boldsymbol{x},\boldsymbol{y})$ the fundamental solution, $q(\boldsymbol{y})$ and $q^*(\boldsymbol{x},\boldsymbol{y})$ the normal derivatives of $p(\boldsymbol{y})$ and $p^*(\boldsymbol{x},\boldsymbol{y})$, and $\boldsymbol{y}$ the field point. For a time factor $e^{-i\omega t}$, we can write $p^*(\boldsymbol{x},\boldsymbol{y})$ and $q^*(\boldsymbol{x},\boldsymbol{y})$ for three-dimensional acoustic problems as

$$p^*(\boldsymbol{x},\boldsymbol{y})=\frac{e^{ikr}}{4\pi r}, \tag{2}$$

$$q^*(\boldsymbol{x},\boldsymbol{y})=-\frac{e^{\mathrm{i}kr}}{4\pi r^2}\,(1-\mathrm{i}kr)\,\frac{\partial r}{\partial n(\boldsymbol{y})}, \tag{3}$$

where i is the imaginary unit, $k=\omega/C_s$ is the wave number, ω is the angular frequency, C_s is the sound speed and r is the distance between $\boldsymbol{x}$ and $\boldsymbol{y}$, i.e., $r=|\boldsymbol{y}-\boldsymbol{x}|$. The boundary conditions (BC) on Γ are given as

$$\text{Dirichlet BC}: p(\boldsymbol{x})=\overline{p}(\boldsymbol{x}) \quad \text{on } \Gamma_p, \tag{4}$$

$$\text{Neumann BC}: q(\boldsymbol{x})=i\rho_0\omega\overline{v}(\boldsymbol{x}) \quad \text{on } \Gamma_v, \tag{5}$$

where ρ_0 is the medium density, $v(\boldsymbol{x})$ is the normal velocity and $\Gamma=\Gamma_p\cup\Gamma_v$. The dash indicates that the value is known.

Eq. (1) which is also referred to as the conventional boundary integral equation (CBIE) in this paper can be utilized to calculate the unknown boundary values. However, BEM based on it fails to yield unique solutions for exterior acoustic problems at the eigenfrequencies of the associated interior problems [3]. These eigenfrequencies are usually called fictitious eigenfrequencies because they have no physical meaning but just arise from the drawback of the boundary integral representation when solving exterior acoustic problems. To overcome this difficulty, two main methods appropriate for practical applications have been proposed over the last several decades, i.e., the combined Helmholtz integral equation formulation (CHIEF) [3] and the Burton–Miller formulation [6]. The latter one which is a linear combination of CBIE and its normal derivative is considered in this paper. It has already been proved that this formulation is more rigorous to circumvent the fictitious eigenfrequency problem than the CHIEF, especially in the high frequency range [4,5].

The derivative of Eq. (1) in the direction of a normal vector $n(\boldsymbol{x})$ is given by

$$c(\boldsymbol{x})q(\boldsymbol{x})+\int_\Gamma \tilde{q}^*(\boldsymbol{x},\boldsymbol{y})p(\boldsymbol{y})\,\mathrm{d}\Gamma(\boldsymbol{y})=\int_\Gamma \tilde{p}^*(\boldsymbol{x},\boldsymbol{y})q(\boldsymbol{y})\,\mathrm{d}\Gamma(\boldsymbol{y}), \tag{6}$$

where $\tilde{(\,)}=\partial(\,)/\partial n(\boldsymbol{x})$ is the normal derivative, and

$$\tilde{p}^*(\boldsymbol{x},\boldsymbol{y})=-\frac{e^{ikr}}{4\pi r^2}(1-ikr)\frac{\partial r}{\partial n(\boldsymbol{x})}, \tag{7}$$

$$\tilde{q}^*(\boldsymbol{x},\boldsymbol{y})=\frac{e^{ikr}}{4\pi r^3}\left[(3-3ikr-k^2r^2)\frac{\partial r}{\partial n(\boldsymbol{x})}\frac{\partial r}{\partial n(\boldsymbol{y})}+(1-ikr)n_i(\boldsymbol{x})n_i(\boldsymbol{y})\right]. \tag{8}$$

In Eq. (8), n_i is the Cartesian component of the vector $n(\boldsymbol{x})$ or $n(\boldsymbol{y})$. The Einstein summation convention is also applied there, so repeated indices imply summation over their range.

Eq. (6) is referred to as the normal derivative boundary integral equation (NDBIE) in this paper. Discretization of Eqs. (1) and (6) by collocation allows us to formulate the system matrices **H** and **G** as

$$H^{ij}=c(\boldsymbol{x}_i)\delta_{ij}+\int_\Gamma q^*(\boldsymbol{x}_i,\boldsymbol{y})\phi_j(\boldsymbol{y})\,\mathrm{d}\Gamma(\boldsymbol{y}), \tag{9}$$

$$G^{ij}=\int_\Gamma p^*(\boldsymbol{x}_i,\boldsymbol{y})\phi_j(\boldsymbol{y})\,\mathrm{d}\Gamma(\boldsymbol{y}), \tag{10}$$