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The use of the constant vector basis functions for the magnetic field integral equation

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ABSTRACT

The magnetic field integral equation (MFIE) is widely used in the analysis of electromagnetic scattering problems for conducting objects. Usually, the MFIE is solved by the method of moments (MoM) using the Rao–Wilton–Glisson (RWG) basis functions. In this paper, a new kind of basis function which is named the piece-wise constant vector basis function is proposed and used to solve the MFIE by MoM. Definition of this kind of basis function is given. The calculation of the impedance matrix entries is presented in detail. This kind of basis function is then used for the solution of the MFIE for electromagnetic scattering problems. The radar cross section (RCS) results and the iterative property of both kinds of basis functions are presented. It is shown that the piece-wise constant vector basis functions give similar RCS results as those of the RWG basis functions. Particularly, when iterative solver is used to solve the resultant linear system, the solution scheme using the piece-wise constant vector basis functions iterates much faster than that using the RWG basis functions.

1. Introduction

Surface integral equations [1,2] are widely used in modelling electromagnetic scattering problems associated with perfect conducting objects. In this method, the proper Green's function are first achieved and used as the integral kernel. Then we integrate on the whole surfaces of the corresponding conducting scatterer. For electromagnetic scattering problems associated with a conducting scatterer in free space, usually, the free space Green's function is used as the integral kernel. The equivalent surface currents are commonly used as the unknown functions. Once the equivalent surface currents are solved, other interesting characteristics such as the radar cross section (RCS) can be calculated easily. If the boundary condition for tangential electric fields are used, then we get the electric field integral equation (EFIE) [3–5] and if the boundary condition for tangential magnetic fields are sued, then we get the magnetic field integral equation (MFIE) [4–6]. Both kinds of integral equations are widely used in modeling the equivalent surface currents for a conducting scatterer. The EFIE is a first-kind Fredholm surface integral equation. The condition number of the corresponding impedance matrix is usually large and the number of iterations is large when solving the corresponding linear system using iterative algorithm. However, The MFIE is a second-kind Fredholm surface integral equation and the number of iterations is usually small when solving the corresponding linear system using iterative algorithm. To solve a surface integral equation, the method of moments (MoM) [4] are widely used. To model arbitrarily shaped three-dimensional surfaces, triangular patches are frequently used to discretize the surface of a conducting scatterer. Then a suitable basis function sets should be used to expand the equivalent surface currents. The most notable basis functions are the Rao–Wilton–Glisson (RWG) basis functions [3]. However, to improve the accuracy of the MFIE, other kinds of basis functions defined on triangular patches are also used recently. In [7,8], the monopolar RWG functions are used for the MFIE for conducting objects with sharp-edges or corners. The $\mathbf{n} \times$ **RWG** basis functions [9–12] and the linear-linear basis functions [13] are reported to give better accurate results than those of the RWG basis functions. Particularly, the Buffa–Christiansen basis functions used in [14] can improve the accuracy of the second-kind Fredholm integral equations greatly. The MFIE is a second-kind Fredholm surface integral equation, and the number of iterations solving the corresponding linear system is usually small. However, the choice of the basis functions also affects the number of iterations. Results reported in the former literatures are usually involving the accuracy problems. However, they usually

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use similar number of iterations when solving the resultant linear system using iterative algorithm as that of the RWG basis functions.

Different from the work in [7–14] to improve the accuracy of the MFIE using new basis functions, we focus on reducing the number of iterations for iterative algorithm when solving the linear systems resulted from the solution of the MFIE. It is shown that although the MFIE is the second-kind Fredholm surface integral equation, the number of iterations solving the corresponding linear system can still be reduced greatly through the use of other basis functions other than the commonly used RWG basis functions. This is realized through the use of a new kind of basis functions which is named the piece-wise constant vector basis functions. Compared with the traditionally used RWG basis functions, the piece-wise constant vector basis functions are orthogonal with each other in the corresponding definition domain. Definitions of this kind basis functions and the construction process in acquiring the piece-wise constant vector basis functions are given in this paper. Besides, we give a detailed discussion on the solution of the MFIE using this basis functions by MoM. Numerical results for electromagnetic scattering analysis from conducting objects with both curved surfaces and with sharp-edges or corners are shown to show the effectivity and efficiency of this new kind of basis functions.

2. The MFIE and the EFIE formulation

The electromagnetic scattering problem by a conducting object is shown in Fig. 1. The time-harmonic electric and magnetic fields are denoted by $\mathbf{E}(\mathbf{r})$ and $\mathbf{H}(\mathbf{r})$, respectively. According to the equivalence principle [15], the problem can be formulated in terms of the equivalent surface electric currents $\mathbf{J}(\mathbf{r})$ defined on the boundary surface $\partial\Omega$ in Fig. 1. Equations governing the equivalent surface currents $\mathbf{J}(\mathbf{r})$ can be formulated by the MFIE which is obtained from the boundary conditions of the magnetic fields $\mathbf{H}(\mathbf{r})$, i.e.,

$$\mathbf{n}(\mathbf{r}) \times \mathbf{H}(\mathbf{r}) = \mathbf{J}(\mathbf{r})|_{\partial\Omega} \tag{1}$$

where in (1) $\mathbf{n}(\mathbf{r})$ is the normal unit vector for $\partial\Omega$. The total magnetic fields $\mathbf{H}(\mathbf{r})$ can be written as the sum of the incident magnetic fields $\mathbf{H}^i(\mathbf{r})$ and the scattered magnetic fields $\mathbf{H}^s(\mathbf{r})$. By the use of the Stratton–Chu formulation, the scattered magnetic field in Ω_{ext} can be expressed as and integral associated with the Green's function in free-space. To express the MFIE compactly, we use an integral operator K which is defined as

$$K(\mathbf{f};\partial\Omega)(\mathbf{r}) = \mathbf{n}(\mathbf{r}) \times p.v.\int_{\partial\Omega} \nabla G(\mathbf{r},\mathbf{r}') \times \mathbf{f}(\mathbf{r}')d\mathbf{r}' \tag{2}$$

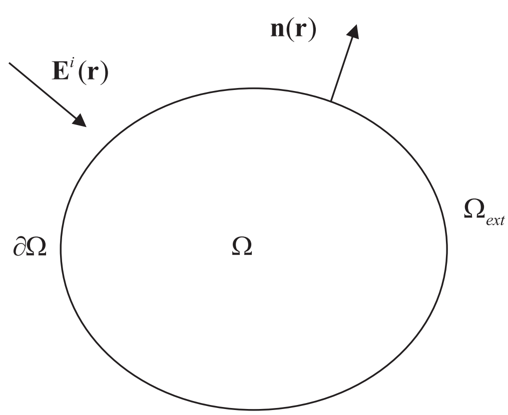

Fig. 1. Electromagnetic scattering of plane waves by a conducting object.

with *p.v.* means the Cauchy principal integral and $G(\mathbf{r},\mathbf{r}') = e^{-jk_0R}/4\pi R$ is the free-space Green's function. $R = |\mathbf{r}-\mathbf{r}'|$ is the distance between the field point $\mathbf{r}$ and the source point $\mathbf{r}'$. Then the MFIE can be written neatly as

$$(1/2)\mathscr{I}(\mathbf{J})(\mathbf{r}) - K(\mathbf{J};\partial\Omega)(\mathbf{r}) = \mathbf{n}(\mathbf{r}) \times \mathbf{H}^i(\mathbf{r}) \tag{3}$$

In (3), $\mathscr{I}$ is the identity operator which maps a vector function into itself.

Similarly, the equivalent surface currents $\mathbf{J}(\mathbf{r})$ can also be formulated by the EFIE which is obtained from the boundary conditions of the electric fields $\mathbf{H}(\mathbf{r})$, i.e.,

$$[\mathbf{E}(\mathbf{r})]_{\tan} = 0|_{\partial\Omega} \tag{4}$$

By the use of the following integral operator T,

$$T(\mathbf{f};\partial\Omega)(\mathbf{r}) = -jk_0\int_{\partial\Omega}[\mathbf{f}(\mathbf{r}') + \frac{1}{k_0^2}\nabla\nabla\cdot{}'\mathbf{f}(\mathbf{r}')]G(\mathbf{r},\mathbf{r}')d\mathbf{r}' \tag{5}$$

the scattered electric fields $\mathbf{E}^s(\mathbf{r})$ can be expressed as

$$\mathbf{E}^s(\mathbf{r}) = T(\eta_0\mathbf{J};\partial\Omega)(\mathbf{r}) \tag{6}$$

In (5), j is the imaginary unit and the free-space wave number $k_0 = \omega\sqrt{\mu_0\varepsilon_0}$. ω is the radial frequency of operation. In (6), the free-space wave impedance $\eta_0 = \sqrt{\mu_0/\varepsilon_0}$.

Since

$$\mathbf{E}(\mathbf{r}) = \mathbf{E}^s(\mathbf{r}) + \mathbf{E}^i(\mathbf{r}) \tag{7}$$

The EFIE can be expressed compactly as

$$\left[T(\eta_0\mathbf{J};\partial\Omega)(\mathbf{r}) = -\mathbf{E}^i(\mathbf{r})\right]\Big|_{\tan} \tag{8}$$

Although both the MFIE and the EFIE can be used to solve the former mentioned electromagnetic scattering problems, numerical characters of these two equations are quite different. It has been observed that EFIE has better accuracy than MFIE [16,17]. Many new kinds of basis functions have been proposed to deal with the inaccuracy problems of the MFIE. On the other hand, MFIE has better convergence rate when solved with an iterative solver. This is because MFIE is a second kind integral equation while EFIE is a first-kind integral equation. Hence, the eigenvalues of EFIE operators tends to cluster around the origin while those of the MFIE operator are shifted away from the origin. However, all of the published papers focus on improving the accuracy of the MFIE. None is associated with the iterative character of the MFIE. It is shown in the following sections of this paper that the iterative character of the MFIE can further be improved greatly through the use of new kind of basis functions.

3. Piece-wise constant vector functions and its application in the MFIE

Characters of integral operator limit the choice of basis functions in method of moments (MoM). For the EFIE, to reduce the high singularity of the gradient-gradient operation in the integral operator T, usually, one gradient operator is transferred to divergence operator on basis functions through vector operation [3]. In this case, basis function has to be the divergence-conforming functions. This kind of function keeps the continuity of the normal component for the expanded vector. The commonly used Rao–Wilton–Glisson (RWG) functions are such kind of divergence-conforming functions. However, different from the EFIE, there is no any vector differential operation on the surface currents $\mathbf{J}(\mathbf{r})$ in the MFIE. Therefore, the continuity condition for the surface currents $\mathbf{J}(\mathbf{r})$ is not so strict as those in the EFIE. In fact, functions that do not impose any normal or tangential continuous conditions can be used to expand the unknown currents in the MFIE. For example, the monopolar RWG basis functions used in [7,8]