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A direct BEM to model the temperature of gradient coils

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ABSTRACT

The temperature of the gradient coils is an important issue in the development of MRI scanners. Gradient coil performance must be maximised within temperature limits imposed by safety and system requirements. Here we present a model that determines the temperature distribution in gradient coils designed using an inverse boundary element method (IBEM). This forward approach is derived by applying a constant boundary element method (BEM) on a steady-state approximation of the heat equation and combined with the stream function associated to an electric current density. It can be used to estimate the temperature distribution, as well as, the location and temperature of hot spots in gradient coils of arbitrary shape. Several examples of the applicability of the proposed BEM model on different coil geometries and thermal characteristics are presented. In order to validate the method, a small prototype *X*-gradient coil was built and tested, and the temperature distribution experimentally measured. It was found to be in a good agreement to the temperature distribution simulated by the proposed numerical approach with a suitable choice of the thermal properties.

1. Introduction

Magnetic resonance imaging (MRI) is a non-invasive technique that relies on the principles of nuclear magnetic resonance (NMR), and can be used for *in vivo* imaging of the human body. MRI is based on the use of well defined and controlled magnetic fields, such as the magnetic field gradients, used to spatially encode the NMR signals from the sample. These field gradients are generated by high currents that flow in coils, which are constructed usually from wire or from cut copper sheets often wound on a cylindrical surface [1], although other geometries can be employed.

In order to provide optimal performance and patient comfort, an ideal gradient coil should have several properties, such as minimal stored magnetic energy, high gradient-to-current ratio, minimal resistance, good field gradient linearity and minimal interaction with the rest of the MRI system and patient. Gradient coil design therefore seeks to find optimal positions for the windings of the coil (or copper sheets) so as to produce fields with the desired spatial dependence and properties [1]. This can be seen as an inverse problem in which several conflicting performance attributes or objectives need to be optimised simultaneously [2].

One of the most important issues in the design and construction of a MRI scanner is the heat dissipation produced by gradient coils, especially with the high electric currents (up to 900 A in some systems) used to produced higher magnetic field. The conduction of high currents through resistive coils leads to considerable Joule heating that can damage the coil or alter behaviour of the rest of the MRI system, leading to a loss of image quality. Gradient coils must therefore be carefully controlled and operated within safe limits.

Some solutions are frequently employed to reduce gradient coil heating, such as the use of special thermally conductive epoxy resins and the integration into the system of cooling mechanisms, where heat is removed by pumping water around the pipes embedded in the former.

The literature relevant to the problem of gradient coil heating, although scarce, has some notable works, such as the one of Chu and Rutt [3], who presented a model based on heat transport theory to describe the spatial average temperature response of cylindrical gradient coils; and where it is highlighted the importance of the location and extent of gradient hot spots in the coil (usually found in regions of high current density) when evaluating gradient heating. This particular problem has also been successfully investigated by While et al. [4] who used a steady-state approximation to model the spatial temperature distribution of cylindrical gradient coils [4], this approach uses heat transfer mechanisms, including Ohmic heating of the coil, as well as conductive, convective and radiative cooling, and insulating

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properties of the coil former. This model has been later included in an optimisation strategy to design cylindrical gradient coils with improved temperature distributions and reduced hot spot temperatures [5,6].

More recently Poole et al. [8] successfully produced thermal simulations of the temperature distribution in gradient coils of arbitrary shape by using elements of discrete differential geometry and the discrete Laplace operator.

On the other hand, over the last two decades new design methods have been developed to improve gradient coil performance and patient comfort. An especially successful group of coil-design techniques are those that incorporate the stream function within an inverse boundary element method (IBEM). The pioneer of this type of approach was Pissanetzky [9]. This technique was also later applied to produce coils with arbitrary geometry [10,11], and extended to higher orders [12,13], proving to be a quite flexible approach allowing the inclusion of new coil features in the design process [14,15].

Additionally, BEM formulations have proved to be an ideally suited approach for solving general heat transfer analysis and especially useful in the description of steady state heat problems [17–19].

Incorporating a BEM formulation of the heat problem in a stream function IBEM for gradient coil design represents then an attractive strategy for the evaluation of the heat dissipation produced by gradient coils.

Here we present a BEM model to determine the temperature distribution in gradient coils produced with the stream function IBEM coil design technique. Unlike previous works [3,6], which are restricted to cylindrical geometries, the proposed method allows the thermal analysis of gradient coils of arbitrary shape.

The model also includes many thermal and electrical coil properties, providing a great tool to predict coil temperature and hot spot locations prior to manufacture.

The structure of this paper is as follows. First, a constant BEM is applied to a steady-state heat equation that defines the gradient heating problem. Next, the idea of stream function of a current density is incorporated in the obtained BEM solution which produces a temperature model that can be included in an IBEM coil design approach. In Section 3, by using the proposed temperature model, numerical simulations have been generated for a set of coils of different thermal properties and geometries. Finally, a small prototype of an X-gradient coil has been built and tested with thermal imaging. Results of the simulations show that the proposed model provides an efficient tool to predict and evaluate coil temperature, hot spots locations and magnitudes for any coil designed with IBEM of arbitrary geometry.

2. Method

In this section, a model for predicting the spatial temperature distribution for a gradient coil designed with an IBEM approach is formulated. First the heat equation that describes the thermal problem is presented, which is subsequently solved using a constant BEM, resulting in the temperature at the coil surface in terms of the current distribution that flows on it. Finally, incorporating the stream function associated to the current distribution, a temperature model suitable for any coil designed with the IBEM approach is obtained.

So let us first consider a copper sheet, Γ, of thickness w, carrying a surface current density $\mathbf{j}$ (driven by a power source), and embedded within two layers of electrically insulating material of the same shape, Ω_1 and Ω_2, as depicted in Fig. 1(a). If a coil centre is defined, the vector $\mathbf{y}$ and $r_c(\mathbf{y})$, $\mathbf{y} \in \Gamma$ are the position and the distance respect to the prescribed origin respectively of a given

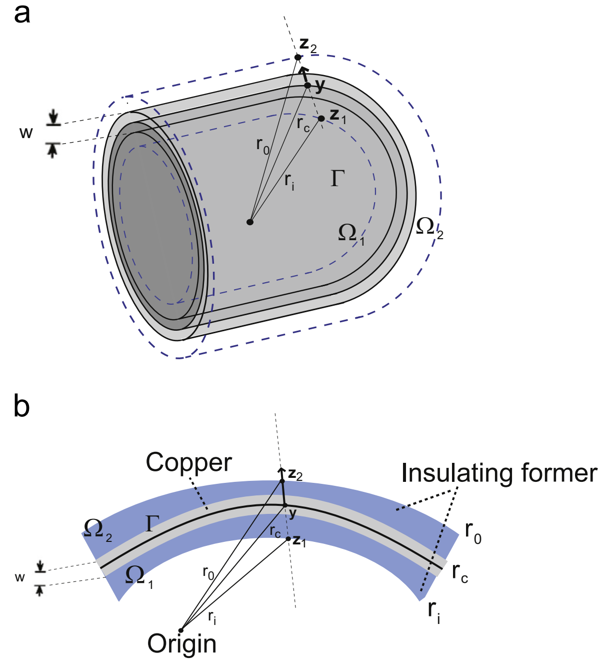

Fig. 1. (a) Coil of arbitrary geometry, the model comprises a thin copper sheet, Γ, embedded in an insulating former bounded by Ω_1 and Ω_2. The cooper sheet carries a surface current density $\mathbf{j}$. The former extends outwards from the copper layer to a distance r_o and inwards to a distance r_i. (b) Schematic showing the cross sectional view of a small portion of the system.

point at the copper layer. The epoxy former extends outwards from $r_c(\mathbf{y})$ to a distance $r_o(\mathbf{z}_2), \mathbf{y} \in \Gamma, \mathbf{z}_2 \in \Omega_2$ and inwards from $r_c(\mathbf{y})$ to $r_i(\mathbf{z}_1), \mathbf{y} \in \Gamma, \mathbf{z}_1 \in \Omega_1$; where $\mathbf{z}_1$ and $\mathbf{z}_2$ are the points where the straight line defined by the normal surface vector at $\mathbf{y}$ intersect Ω_1 and Ω_2 respectively.

A suitable representation of the temperature above ambient, T^*, of the copper sheet can be produced by a heat equation in which the dissipation of heat throughout the object is governed by the Laplacian of T^* and heat is lost from the object by a cooling term proportional to T^*. An Ohmic heating term must be also included to take into account the power dissipated as heat energy due to $\mathbf{j}$. By assuming isotropic and uniform thermal conductivity, ignoring any non-linear effects and considering the steady-state equilibrium, the heat equation that describes this problem can be written as [7]

$$\nabla^2 T^*(\mathbf{x}) - \frac{h_t}{k_e w} T^*(\mathbf{x}) = -\frac{\rho_r}{k_e w^2} \mathbf{j}(\mathbf{x}) \cdot \mathbf{j}(\mathbf{x}), \tag{1}$$

where h_t is the total heat transfer coefficient, k_e is the effective thermal conductivity and ρ_r is the resistivity of the copper.[1] Details regarding the calculation of both coefficients can be found in [3,7].

Eq. (1) is often referred to as a screened Poisson equation [23], which, for sake of simplicity, can be written as

$$\nabla^2 T^*(\mathbf{x}) - K^2 T^*(\mathbf{x}) = -c(\mathbf{x}), \quad \mathbf{x} \in \Gamma. \tag{2}$$

where

$$K = \sqrt{\frac{h_t}{k_e w}}, \tag{3}$$

[1] Many material properties involved in this approach, such as the conductivity, are function of the temperature. The inclusion of these couplings would lead to an interesting but much more mathematically complex problem. In this work constant material properties are considered, as this approximation has proven to produce accurate solutions in the calculation of temperature distributions of gradient coil [3,7].