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Modeling of fluid flow through fractured porous media by a single boundary integral equation

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ABSTRACT

The objective of this work is to provide theoretical materials for modelling two-dimensional fluid flow through an anisotropic porous medium containing intersecting curved fractures. These theoretical developments are suitable for numerical simulations using boundary element method and thus present a great advantage in mesh generation term comparing to finite volume discretization approaches when dealing with high fracture density and infinite configuration. The flow is modelled by Darcy's law in matrix and Poiseuille's law in fractures. The mass conservation equations, at a point on the fracture and an intersection point between fractures in the presence of a source or a sink, are derived explicitly. A single boundary integral equation is developed to describe the fluid flow through both porous media and fractures, i.e. the whole domain, which includes particularly the mass balance condition at intersection between fractures. Numerical simulations are performed to show the efficiency of this proposed theoretical formulation for high crack density.

1. Introduction

Modelling of fluid flow within porous geological formations containing a high fracture density is a subject of interest, as well as one of the most challenging problems for many application fields such as petroleum engineering, groundwater hydrology, geothermal energy, etc. To deal with real complex problems and to obtain local flow information, the numerical methods impose naturally. The domain discretization methods, such as finite element method (FEM) [1,2], finite volume method (FVM) [3,4], raise a major difficulty to generate an appropriate mesh for a domain containing numerous randomly distributed fractures. The methods, based on boundary integral equations (BEM), are computationally efficient and accurate for modelling the fluid flow in porous media thank to its advantage of reduction of problem dimension [5,6]. However, the classical BEM exhibits mathematical degeneracy for domain containing discontinuities. Alternative techniques are proposed for overcoming this difficulty such as Accelerated Perturbation BEM [7], Multi-domain Dual Reciprocity Method [8,9], Green element method [10,11], and Multi-region BEM [25–27]. However, these advanced techniques require a complicated numerical implementation in comparison with the standard BEM and also become inefficient for high fracture density.

Numerical developments, based on the symmetric Galerkin boundary element method, are presented by Rungamornrat and Wheeler [12] and Rungamornrat [13] for a full consideration of fluid flow through heterogeneous and anisotropic porous media containing non-conductive surface of discontinuity. The nonhomogeneous media consist of several subdomains with different properties. In those works, weak singular weak-form equations are established for pressure and its derivatives, i.e. flux flow. These interesting works are thus ready for application fields of fluid flow within porous media containing fractures or faults that act as barriers to flow.

As a matter of fact, a fracture is generally much more conductive than the surrounding matrix, i.e. fluid flow presents both in the porous matrix and the fractures system. A system of boundary integral equations for flow in fractured porous media was first introduced by Rasmussen et al. [14], in which the matrix and fractures are treated as separate systems having a common interface made up of the fracture boundaries that are contained in the matrix. Therefore, this system is constituted by four equations: two boundary integral equations for the matrix and the fractures, and two conditions for pressure and velocity at fracture–matrix interfaces. Numerical computation requires a fine mesh to calculate accurately the difference between the unknowns at the collocation points on the two sides of the fractures. This difficulty is partially overcome in Lough et al. [15] by modelling the fractures as planar sources within the matrix. Nevertheless, this new model also consists in coupling two integral equations between matrix and fractures with the added unknowns of

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source strength on the planar fractures. This numerical procedure is used by Teimoori et al. [16,17] to simulate the naturally fractured reservoir. The integral equation has been also used to study the heat extraction by circulating water in a fracture embedded geothermal reservoir [18]. This approach allows eliminating the discretization of reservoir and takes into account multi-dimensional effects compared to theoretical solutions. The complicated numerical implementation of this method does not show the advantage of BEM over the FEM and FVM. This could be a reason why BEM community is less active than FEM and FVM one when dealing with fluid flow through porous media in the presence of high crack density.

The present work aims to develope a single boundary integral equation (BIE) in order to describe fluid flow through two-dimensional fractured porous media that allows dealing with high crack density. As a matter of fact, the fractures distributed within the porous media are obviously three-dimensional configuration. However, three-dimensional problems could be simplified to two-dimensional ones in several applications such as Excavation Damaged Zone around a tunnel or a borehole [19], fault zone [20], etc. In the two-dimensional consideration, we make an assumption that fracture has zero thickness and an infinitely transversal permeability, i.e. there is no pressure jump across the fracture. The discontinuity of fluid flow across the fracture is related to fluid flow within the fracture by the mass balance equation at a point on the fracture. Mass exchanges between fractures and porous matrix, at intersection between fractures, in the presence of source and sink points, are explicitly formulated based on the recent works [21–24]. Considering the fractures as the internal boundary, the BIE is written for fluid flow within the porous matrix. This equation links to fluid flow by means of fracture by the boundary condition on the internal boundary, i.e. the pressure and flow at the fracture–matrix interfaces. The mass exchange between matrix and fracture, as well as the fluid flow constitutive law within the fracture result in a single BIE that describes fluid flow within whole fractured porous media. This BIE presents the pressure field as a function of the pressure and the flow on boundary of domain and the pressure on the fracture system. It is worth recalling that the condition at the intersection between fractures, in numerical simulation by whatever methods, has been ignored in the literature by the lack of an explicit mass conservation at this point. The development procedure of BIE allows integrating this mass conservation at intersection points between fractures. This cancel the singularity at these points.

A single BIE, developed to describe fluid flow within both the fractures and embedding porous matrix, presents a great advantage in numerical implementation in comparison to a system of four equations [14–17]. A quick numerical resolution based on collocation method is performed in order to validate and show the efficiency of the proposed BIE.

2. Governing equations

Considering a homogeneous medium Ω embedding a set of n interconnected fractures $\Gamma = \cup \Gamma_i$ (i=1,n) (Fig. 1). In mathematical model, the fracture, supposed to have zero-thickness, is modelled by a smooth function $\underline{z}^i(s)$ of the curvilinear abscise s. This function represents the positions of fracture i within the domain. The porous matrix corresponds to $\Omega - \Gamma$. There are m sources or sinks located at points of coordinates $\underline{x}_k$ (k=1,m) within Ω with corresponding intensities q_k. These sources or sinks can be allocated within the porous matrix (m_p points), on fractures (m_f points) or at fractures intersection points (m_s points), thus $m = m_p + m_f + m_s$. S denotes the set of intersection points between fractures, fracture endpoints and source or sink points on fractures.

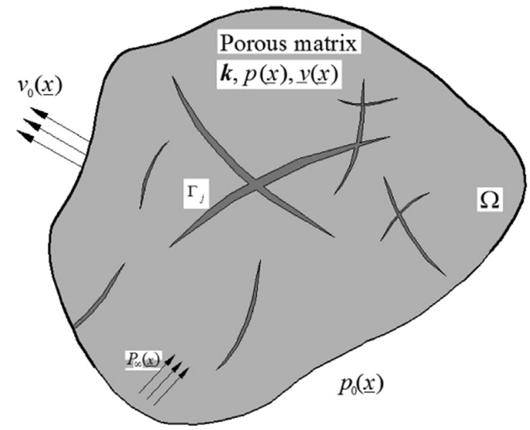

Fig. 1. Fluid flow within fractured porous media.

Fluid flow is governed by Darcy's law (1) in the matrix and Poiseuille's law (2) in the fractures:

$$\forall \underline{x} \in \Omega - \Gamma, \quad \underline{v}(\underline{x}) = -\frac{\boldsymbol{k}(\underline{x})}{\mu} \nabla p(\underline{x}) \tag{1}$$

$$\forall s \in \Gamma, \quad q(s) = -c(s) \partial_s p \tag{2}$$

where v(x), p(x) are the fluid velocity and pressure fields within the porous matrix, respectively; k(x) the matrix's intrinsic permeability tensor; q the fluid flow in the fracture and c the fracture conductivity.

The fracture conductivity is determined commonly by cubical law as $c = e^3/(12 f \mu)$, in which e is the fracture aperture, μ the dynamic viscosity of fluid and f the roughness factor of fracture surfaces [28]. From geometrical point of view, the fracture has zero thickness to be modelled as a one dimensional curve within two dimensional spaces. However, in the physical model, there exists an aperture of facture, i.e. the fracture conductivity according to Poiseuille's law (2).

The continuity equation of fluid in the porous matrix reads:

$$\forall \underline{x} \in \Omega - \Gamma, \quad \nabla . \underline{v}(\underline{x}) + \sum_{k=1}^{m_p} q_k \delta(\underline{x} - \underline{x}_k) = 0 \tag{3}$$

where δ represents the *Dirac* distribution.

The mass conservation for flow within the fracture, excluding the fractures intersection points and source or sink points, is written as [3,4,21,24]

$$\forall s \in \Gamma - S, \quad [[\underline{v}(\underline{z})]] \cdot \underline{n}(s) + \partial_s q = 0 \tag{4}$$

where $\underline{z}$ is the point on the factures at abscise s, $\underline{n}(s)$ the normal unit vector to the fracture oriented from Γ^- to Γ^+ and $[[\underline{v}(\underline{z})]] = \underline{v}^+(\underline{z}) - \underline{v}^-(\underline{z})$ is the velocity jump across the fractures.

For the mass balance condition at the intersection point having no source or sink, Pouya and Vu [21] showed that the sum of outgoing fluid flow vanishes, i.e.$\sum_i q_i^0 = 0$, where q_i^0 is the outflow in the fracture branch i from the intersection point. Their demonstration method is used to derive the mass conservation expressions within the fractures or at fracture intersection points including sources or sinks. Considering a small domain D surrounding a fracture intersection point $\underline{z}$ where there is the presence of a source with intensity $q^s(\underline{z})$. There are $I-1$ fractures come together at this point. Let us now replace the source by a fictitious fracture Γ_I meeting with other fractures at $\underline{z}$. The flow within Γ_I is constant and equal to $q^s = q^I$ (Fig. 2a), i.e. there is no exchange between the fictitious fracture and the porous matrix.