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A model-integrated localized collocation meshless method for large scale three-dimensional heat transfer problems

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ABSTRACT

We present a Model Integrated Meshless Solver (MIMS) tailored to solve practical large-scale industrial problems. This is accomplished by developing a robust meshless technique as well as a comprehensive model generation procedure. By closely integrating the model generation process into the overall solution methodology, the presented techniques are able to fully exploit the strengths of the meshless approach to achieve levels of automation, stability, and accuracy currently unseen in the area of engineering analysis. Specifically, MIMS implements a blended meshless solution approach which utilizes a variety of shape functions to obtain a stable and accurate iteration process. This solution approach is then integrated with a newly developed, highly adaptive model generation process which employs a quaternary triangular surface discretization for the boundary, a binary-subdivision discretization for the interior, and a unique shadow layer discretization for near-boundary regions. Together, these discretization techniques are able to achieve directionally independent, automatic refinement of the underlying model, allowing the method to generate accurate solutions without the need for intermediate human involvement. In addition, by coupling the model generation with the solution process, the presented method is able to address the issue of ill-constructed geometric input such as small features, poorly formed faces, and other such pathologies often generated from solid models in the course of design and in the end to provide an intuitive, yet powerful approach to solving modern engineering analysis problems.

1. Introduction

There are numerous numerical techniques capable of solving partial differential equations within the framework of engineering analysis, and, most commonly, industrially relevant numerical techniques rely on structured connectivity between nodes to define the problem geometry. As such, these techniques are generally classified as mesh-based techniques (for their use of a structured connectivity mesh). Finite difference, finite element, and finite volume methods may all be broadly placed into this category of solution techniques. Although mesh-based methods have proven themselves capable for a wide range of problem domains, there is still the undesirable responsibility of having to define a connectivity within the solution domain. Despite efforts to automate the mesh generation process, a considerable amount of time and human effort is still spent preparing and meshing the computational model when presented with a problem consisting of complex geometry.

In an attempt to eliminate the need for the underlying nodal connectivity, some researchers have turned to the area of meshless and mesh-reduction methods [1–7]. These methods, which seek to replace the underlying structured connectivity with an unstructured interpolation scheme, have shown considerable promise in many application areas. However, they have failed, as of yet, to provide competition to more conventional mesh-based approaches when applied to real-world, industrially relevant applications. This may largely be attributed to the relative youth of the field; however, it may also be caused by the focus given by many meshless methods researchers to generating new meshless techniques, while failing to address the underlying cause for concern in model discretization. This failure to address the underlying issues of mesh generation has resulted in a general lack of practicality of the field and has largely relegated meshless methods, at least for the near future, to academic endeavors and specialized application domains.

Despite the inability of meshless methods to thus far outcompete more traditional mesh-based techniques, there have been considerable advances within the field. Modern meshless implementations are generally at least as efficient as unstructured mesh-based techniques and therefore have reached the solution potential of more traditional approaches. It is for this reason that the focus of this paper is to present a meshless methodology and solution approach which builds upon current meshless research whose aim is to allow analysis of industrially relevant problems.

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At the core of this approach is a collocation-based meshless method [7–24] which has been developed to be both robust as well as accurate under a variety of nodal configurations. This is accomplished through the use of a unique blend of existing meshless shape functions and development of a point distribution process specifically designed to take advantage of the liberties granted by the meshless techniques. Through close integration of the model generation and solution process, the Model Integrated Meshless Solver (MIMS) is able to specifically address the underlying issues which make mesh generation such a time consuming and tedious process. In addition, this integration allows for a highly adaptive and robust system capable of efficiently evolving both the solution, as well as the underlying discretization with no human interaction.

The organization of this paper is as follows, Section 2 will present an overview of the localized collocation meshless method (LCMM) implementation used throughout this paper. Although full details are omitted for brevity (interested readers may follow pertinent references in that section), a general outline of the goal and procedures will be presented, allowing the reader to appreciate the remaining work. Section 3 will present the model generation process that has been developed as part of this effort, and represents the major advancement to the state-of-the-art in meshless research. Together with the adaptive refinement process presented in Section 4, the model generation procedure is what allows the MIMS method to achieve levels of robustness and accuracy competitive with currently available analysis packages. Once all necessary components of the MIMS method have been described, Section 5 will then present an important implementation detail in the shape function selection process. Finally, an industrial case study will be performed to illustrate a comparative example to a commercially available solution package and conclusions will be presented.

2. Meshless implementation overview

Most papers regarding meshless methods focus primarily on particular implementation details (shape functions, interpolating kernels, governing equations, etc.) and seek to demonstrate how the described techniques exhibit solution accuracy, speed, and robustness. However, as has already been pointed out, there has been little focus on the model generation procedures and as such, that will be the primary focus of this paper. That being said, it is still important to provide an overview of the meshless implementation used in MIMS and how it has been designed to integrate within the overall framework.

When approaching meshless methods from a practical point of view, it should be clear that there are many different techniques which may be utilized to obtain a capable method. However, the goal of this research is not simply to develop a capable method, but to develop a general solution methodology able to compete against more traditional methods such as finite element and finite volume. To accomplish this, we must address two distinct problems with mesh-based techniques, (1) generating the necessary meshes is a time consuming process involving considerable human interaction, and (2) solution quality can be highly dependent on the quality of the mesh. Interestingly enough, it is not the direct elimination of the underlying mesh which allows meshless methods to solve these issues (indeed, few would disagree that solution quality is still dependent on the quality of the underlying nodal distribution with meshless methods), instead the liberties that are granted during point distribution are what facilitates the ease of use and solution generality. This is a point often missed, and consequently, so is the fact that the underlying point distribution techniques are arguably as important as the methods themselves. It is the point generation liberties granted by meshless methods that provide their advantage over mesh-based techniques.

To best take advantage of these so-called point generation liberties, the authors have chosen to use a collocation-based meshless approach to serve as the foundation of the solution mechanism [7–24]. Collocation was chosen for several reasons, the first being that it is a point-based approach, and as such, can be applied directly to a solution domain without special consideration for boundary condition application, as is often the case when utilizing a non-interpolating approximation [1]. The second reason for choosing collocation was due to the fact that collocation techniques can be formulated such that their computational time and memory requirements are kept at a minimum (due to the local nature of the formulations). The final reason is that collocation allows for the use of a variety of interpolation schemes in order to develop the underlying shape functions for field and derivative evaluation. Understanding that the meshless method will utilize collocation to formulate the updating scheme for the governing equations, the next step was to decide on appropriate shape functions to represent the underlying solution field and its derivatives. It is in this respect that the current method departs from current techniques in that no single interpolating method is used to construct the necessary shape functions. Instead, a blend of moving least squares [4], radial basis function interpolation [5–11], and virtual finite differencing [18–24] is utilized to obtain a method that is both stable and accurate. This departure allows for a method which is not married to any particular interpolation scheme, and as such may take advantage of the relative strengths and weaknesses of each technique. We now briefly review all three methods as they apply to transient heat conduction.

2.1. The localized collocation meshless method (LCMM) framework

The meshless formulation begins by defining a set of data centers, *NC*, comprised of points on the boundary, *NB*, and points on the interior, *NI*. These data centers will serve as collocation points for the localized expansion of the different field variables in the domain, Ω, and on the boundary, Γ (see Fig. 1). The essential difference between boundary points and internal points is simply that boundary conditions will be applied at the first while governing equations will be applied at the last. We apply the LCMM to the diffusion equation for the field variable, ϕ, in a generalized coordinate system, x, time, t, and the diffusion coefficient, κ, will be taken into consideration as the governing equation valid in the domain, Ω, as follows:

$$\frac{\partial\phi}{\partial t}(x,t)=\kappa\nabla^2\phi(x,t) \quad (1)$$

In addition, a set of generalized boundary conditions for the variable, ϕ, on the boundary, Γ, is given by

$$\hat{\beta}_1\frac{\partial\phi}{\partial n}+\hat{\beta}_2\phi=\hat{\beta}_3 \quad (2)$$

where $\hat{\beta}_1$, $\hat{\beta}_2$, and $\hat{\beta}_3$ are the imposed coefficients of (x,t) that dictate the boundary condition type and constrain values. A linear

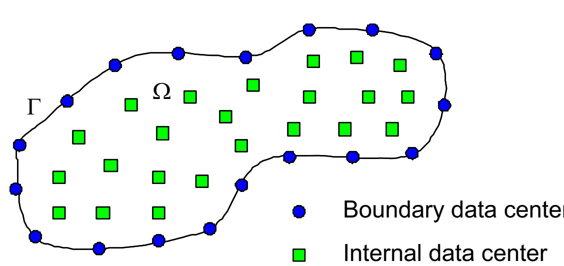

Fig. 1. Scattered point distribution in a generalized domain.