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Dynamic analysis of elastoplastic models considering combined formulations of the time-domain boundary element method

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ABSTRACT

In this work, the combination of the time-domain boundary element method with other numerical procedures is discussed, in order to analyze dynamic elastoplastic models. Since the computation of stress components taking into account time-dependent fundamental solutions is quite elaborate and computationally demanding, this work reviews this procedure considering combined methodologies to evaluate the stress fields. In this context, coupled and hybrid formulations are discussed here. Taking into account coupling procedures, standard, iterative and direct coupling techniques are presented, being the time-domain boundary element method coupled with the domain boundary element method and with the finite element method. For the hybrid formulation, some concepts of the finite element method are introduced into the time-domain boundary element method, avoiding in this way most of its inconveniences considering the dynamic analysis of inelastic models. At the end of the paper, numerical examples are presented, illustrating the accuracy and potentialities of the discussed methodologies.

1. Introduction

The numerical simulation of arbitrarily shaped continuous bodies, subjected to transient loads and non-linear constitutive relations, remains, despite much effort and progress over the last decades, a challenging area of research. In most cases, discrete techniques, such as the finite element method (FEM) and the boundary element method (BEM) have been employed and continuously developed with respect to accuracy and efficiency.

There are three different formulations of dynamic elastoplastic analysis using the BEM that are commonly used, namely: the domain boundary element formulation (D-BEM); the dual reciprocity boundary element formulation (DR-BEM); and the time-domain boundary element formulation (TD-BEM). In the first two formulations, static fundamental solutions are employed and domain integrals, related to inertial and initial stress (or initial strain) terms, are considered. Maintenance of the inertial domain integral generates the D-BEM, whereas its transformation into a boundary integral by adopting suitable approximations for acceleration components (dual reciprocity technique) generates the DR-BEM. In both cases, once the numerical system of equations is established considering the correspondent spatial discretization, a time-marching scheme is introduced (usually the Houbolt method [1]), allowing the advance of the solution on time (temporal discretization). In the third formulation (TD-BEM), time-dependent fundamental solutions are considered and domain discretizations are restricted to regions where inelastic terms are expected to occur. Although, in this case, the causality property is well represented and accurate results are expected, the TD-BEM is very demanding in computational terms, and overly complex from a mathematical standpoint. For more details concerning each BEM formulation described above, the reader is referred to [2–6]; for a review of boundary element methodologies applied to the solution of inelastic dynamic problems, the work of Beskos [7] and Hatzigeorgiou and Beskos [8] may be referred.

In spite of its complexity, the TD-BEM is a very interesting numerical tool, since it allows models with high stress concentration and/or with infinite physical extension to be analyzed in a very consistent way. Most of the drawbacks of this formulation, considering the analysis of inelastic problems, are related to the stress state evaluation. As described by Carrer and Mansur [9], the computation of stress components taking into account time-dependent fundamental solutions is elaborate and arduous to implement (however, alternative approaches have been presented, which describe more feasible techniques [10]). In the present work, these calculations are reviewed and stresses are computed taking into account combined methodologies.

It did not take long until some researchers, seeking to avoid the limitations of the TD-BEM and to profit from its advantages, started to combine the TD-BEM with other numerical methodologies. Nowadays, several publications on the topic are available and, in most of them, BEM–FEM coupling procedures are discussed [11–16]. In these coupled approaches, the FEM is usually employed to model sub-domains where non-linear behavior is expected to occur and the TD-BEM is applied to

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model linear sub-domains or simply to act as a transmitting (or non-reflecting) boundary. Recently, special attention has been devoted to the formulation of more flexible coupling techniques, allowing independent time and/or space discretizations to be considered [17–20]. The coupling of the TD-BEM with other boundary element methodologies has also been implemented, following similar guidelines [21,22].

In this work, three coupling formulations are discussed, namely the standard, the iterative and the direct coupling formulation. In a standard coupling approach, the algorithm for constructing unified equations is highly complicated when compared with that for the FEM or the BEM considered independently. In order to overcome this inconvenience, iterative coupling approaches have been developed [23–31]. In these coupling algorithms, each FEM and BEM sub-domains are considered separately, and successive renewals of variables on the interface of both sub-domains are preformed until the final convergence is achieved. Presently, domain decomposition coupling approaches applied to nonlinear models can be find in the literature, mostly in static analyses. Elleithy et al. [23], for instance, extended the sequential Neumann iterative coupling procedure to consider elastoplasticity and presented a brief review of previous iterative algorithms which were applied to linear analyses: Gerstle et al. [24], Perera et al. [25] and Kamiya and Iwase [26] utilized the conjugate gradient method, the Schur complement and condensation to renew the unknowns at interfaces; Kamiya et al. [27] employed the Schwarz Neumann–Neumann and Schwarz Dirichlet–Neumann renewal schemes; Lin et al. [28] and Feng and Owen [29] discussed a sequential form of the Schwarz Dirichlet–Neumann method; Elleithy and Al-Gahtani [30] presented an overlapping iterative domain decomposition method for coupling the FEM and BEM; etc. Recently, Soares et al. [17,31] and Phansri et al. [32] employed an iterative coupling technique to deal with non-linear dynamic models. Direct coupling procedures (i.e., considering no iterative process) can also be applied to combine the BEM and the FEM, still taking into account a domain decomposition context. This can be carried out by considering explicit time marching techniques within the FEM sub-domain. This technique was presented by Rizo and Wang [33] considering linear models and a staggered solution approach, and by Soares et al. [18] and Romero et al. [34] taking into account dynamic elastoplastic models and the Green–Newmark time-marching technique [35–37]. Similar direct approaches have been presented considering the central difference method [38].

Alternatively to coupling procedures, hybrid formulations can also be developed. In hybrid formulations, one numerical method is modified by the introduction of features related to another numerical method. In this work, a hybrid BEM–FEM formulation is discussed, where the tractions and the displacements of the model are evaluated taking into account the TD-BEM, and the stresses of the model are computed based on finite element procedures. This hybrid technique avoids most of the TD-BEM inconveniences considering the dynamic analysis of inelastic models [39].

The paper is organized as follows: in Section 2, the governing equations for the dynamic elastoplastic model are presented, and, in Section 3, the standard TD-BEM formulation is briefly discussed, as well as some domain discretization techniques (namely the D-BEM and the FEM); in Section 4, coupling techniques are described and, in Section 5, the hybrid BEM–FEM formulation is discussed; at the end of the paper (Section 6), two numerical applications are considered, illustrating the accuracy and potentialities of the different methodologies.

2. Governing equations

The basic equations related to the dynamic modeling of elastoplastic materials are given by:

$$\sigma_{ij,j} - \rho \ddot{u}_i + b_i = 0 \tag{1}$$

$$d\sigma_{ij} = D^{ep}_{ijkl} d\varepsilon_{kl} \tag{2}$$

$$d\varepsilon_{ij} = \tfrac{1}{2}(du_{i,j} + du_{j,i}) \tag{3}$$

where Eq. (1) is the equilibrium equation and Eqs. (2) and (3) stand for incremental relations. The Cauchy stress, using the usual indicial notation for Cartesian axes, is represented by σ_{ij}, and u_i and b_i stand for displacement and body force distribution components, respectively. Inferior commas and overdots indicate partial space and time derivatives, respectively, and ρ stands for the mass density. Eq. (2) is the constitutive law, written incrementally. The incremental strain components $d\varepsilon_{ij}$ are defined in the usual way from the displacements, as described by Eq. (3). In Eq. (2), D^{ep}_{ijkl} is a tangential tensor defined by suitable state variables and the direction of the increment. Within the context of associated isotropic work hardening theory, the tangent constitutive tensor is defined as:

$$D^{ep}_{ijkl} = D_{ijkl} - (1/\gamma) D_{ijmn} a_{mn} a_{op} D_{opkl} \tag{4}$$

where

$$D_{ijkl} = 2\mu\nu/(1-2\nu)\delta_{ij}\delta_{kl} + \mu(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk}) \tag{5a}$$

$$a_{kl} = \partial\overline{\sigma}/\partial\sigma_{kl} \tag{5b}$$

$$\gamma = a_{ij} D_{ijkl} a_{kl} + H \tag{5c}$$

$$H = \partial\sigma_0/\partial\overline{\varepsilon}^p \tag{5d}$$

In Eqs (5), $\overline{\sigma}$ and $\overline{\varepsilon}^p$ are the equivalent (or effective) stress and plastic strain, respectively; σ_0 is the uniaxial yield stress; H is the plastic-hardening modulus (the current slope of the uniaxial plastic stress–strain curve) and μ and ν stand for the shear modulus and the Poisson ratio, respectively. Recall that in case of von Mises isotropic strain-hardening material, the tensor of incremental elastoplastic material moduli takes the form $D^{ep}_{ijkl} = D_{ijkl} - (3\mu/(\sigma_0^2(1+H/3)))s_{ij}s_{kl}$, where $s_{ij} = \sigma_{ij} - (1/3)\delta_{ij}\sigma_{kk}$ is the stress deviator; and for the case of a perfectly plastic material $H=0$. In case of elastic analyses, the Cauchy stresses can be defined by $\sigma_{ij} = D_{ijkl}\varepsilon_{kl}$, where D_{ijkl} (see Eq. (5a)) is the elastic constitutive tensor (this linear relation is a particular case of Eq. (2)).

For the initial stress formulation, it is convenient to define a fictitious "elastic" stress increment as follows:

$$d\sigma^e_{ij} = D_{ijkl} d\varepsilon_{kl} \tag{6}$$

and to rewrite Eq. (2) as indicated below

$$d\sigma_{ij} = d\sigma^e_{ij} - d\sigma^p_{ij} \tag{7}$$

where the initial stress increments are computed by

$$d\sigma^p_{ij} = (1/\gamma) D_{ijmn} a_{mn} a_{kl} d\sigma^e_{kl} \tag{8}$$

In addition to Eqs. (1)–(8), boundary and initial conditions have to be prescribed in order to completely define the problem. They are given as follows:

(i) Boundary conditions ($t \geq 0$, $X \in \Gamma$ where $\Gamma = \Gamma_u \cup \Gamma_p$):

$$u_i = \overline{u}_i \quad \text{for} \quad X \in \Gamma_u \tag{9a}$$

$$p_i = \sigma_{ij} n_j = \overline{p}_i \quad \text{for} \quad X \in \Gamma_p \tag{9b}$$

(ii) Initial conditions ($t=0$, $X \in \Omega$):

$$u_i = \overline{u}^0_i \tag{10a}$$

$$\dot{u}_i = \dot{\overline{u}}^0_i \tag{10b}$$

where the prescribed values are indicated by over bars and p_i stands for traction components along the boundary whose unit