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Fully enriched weight functions in mesh-free methods for the analysis of linear elastic fracture mechanics problems

Reza Namakian [a], Hossein M. Shodja [a,b,*], Mohammad Mashayekhi [a]

[a] Center of Excellence in Structures and Earthquake Engineering, Department of Civil Engineering, Sharif University of Technology, P.O. Box 11155-9313, Tehran, Iran
[b] Institute for Nanoscience and Nanotechnology, Sharif University of Technology, P.O. Box 11155-9313, Tehran, Iran

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ABSTRACT

The so-called enriched weight functions (EWFs) are utilized in mesh-free methods (MMs) to solve linear elastic fracture mechanics (LEFM) problems; the following issues are of concern: convergence behavior; sufficiency of EWFs to capture singular fields around the crack-tip; and the preservation of the *J*-integral path-independency. EWFs prove useful in conjunction with the moving least square reproducing kernel method (MLSRKM); for this purpose, both EWFs and MLSRKM are modified. Since EWFs are not truly representative of the near-tip solution, fully EWFs (FEWFs) are introduced. Finally, some descriptive examples address the aforementioned concerns and the accuracy and efficacy of the proposed technique.

1. Introduction

Mesh-free methods (MMs) have shown a great success in solving boundary value problems for various fields of computational mechanics. These methods were first introduced as smooth particle hydrodynamics (SPH) by Lucy [1] and Gingold and Monoghan [2] to solve problems in astrophysics. Then, SPH was applied to solid mechanics for the first time by Libersky et al. [3]. However, it was observed that SPH had some deficiencies such as tensile instability and incapability to preserve reproducing (completeness or consistency) conditions, specifically along boundaries [4]. Afterwards, Belytschko et al. [5] applied the moving least square method (MLSM), which was introduced by Lancaster and Salkauskas [6], as the first mesh-free Galerkin method to solve governing partial differential equations for solids in a global weak form and called it the element-free Galerkin method (EFGM). One year later, Liu et al. [7,8] alleviated the tensile instability and consistency shortcomings in the relations associated with SPH via introducing a correction term into this method and called it the reproducing kernel particle method (RKPM) that is founded on the wavelet theory [9]. More recently, some kinds of higher order reproducing forms of this method, such as the gradient reproducing kernel particle method (GRKPM) [10–13], have been developed. Although RKPM is very similar to EFGM in formula, they have different origins. Owing to this fact, Liu et al. [14] presented the moving least square reproducing kernel method (MLSRKM) unifying RKPM and EFGM, as two special discretized cases, under one umbrella. One of the generalized approaches to MLSRKM has been lately presented in [15]. MLSRKM is adopted here due to its stability in preserving the consistency conditions for a discretized domain due to the shifted and scaled basis usage in comparison with MLSM [16]. Furthermore, it has been observed that variable volumes of particles or quadrature weights in MLSRK approach are more accurate than the uniform weights used in MLSM, especially along boundaries [17]. References [4,17–22] provide further information about various types of MMs and their properties.

MMs can simply provide smooth and global approximations to the desired order such that they are introduced as more eligible candidates to satisfy equilibrium equations in a weak form [18] and yield continuous stress field throughout the entire domain without any post-processing compared to the Finite Element Method (FEM). In addition, MMs enjoy flexibility in handling mesh sensitive problems [17,20]. As their name implies, MMs are free from mesh, and presumed connectivity for a discretized domain is not required anymore. Thereupon, they can be

* Corresponding author at: Center of Excellence in Structures and Earthquake Engineering, Department of Civil Engineering, Sharif University of Technology, P.O. Box 11155-9313, Tehran, Iran. Tel.: +98 21 66164209; fax: +98 21 66014828.
E-mail addresses: r.namakian@gmail.com (R. Namakian), shodja@sharif.edu (H.M. Shodja), mashayekhimohammad23@gmail.com (M. Mashayekhi).

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considered instrumental in modeling discontinuities like propagating cracks; because, contrary to FEM, there is no need for a burdensome task of remeshing due to the simulation of the crack growth. Another beneficial property of MMs, which is the main topic of this paper, is capability of including a prior knowledge of a local solution, such as singular field around the crack-tip in linear elastic fracture mechanics (LEFM) problems, known as enrichment techniques.

The enrichment techniques are categorized as two main groups in computational approaches. The first one consists of extrinsic enrichment techniques basically resulting in the increase in the number of unknowns and consequently high computational cost of solution due to a broader band-width in the stiffness matrix [21]. The possible ill-conditioning in the linear system of equations could be another problem of the extrinsic enrichments [23]. The problems associated with these enrichments can be heightened in 3D cases. As an example of the extrinsic enrichment techniques, the extended element-free Galerkin method (XEFG) first proposed in the context of LEFM [24] can be mentioned. Later, this approach was developed to deal with problems involving nonlinear materials and cohesive cracks [25–28]. However, due to the heavily overlapping shape functions in the presented methods [25–28], crack closure at the crack-tip should be enforced through a tip enrichment technique. Therefore, some alternative methods were suggested [29–31]; though, these approaches turn out to be complicated for the analysis of situations where the crack front is curved in three dimensions [32]. In addition, the accuracy of the methods [25–31] has not been clearly shown through the examination of the fracture parameters [32]. Although the aforementioned mesh-free methods which take advantage of extrinsic enrichments require the crack path continuity; in the cracking particles method [26,27,33,34] there is no need to have continuous crack path. Consequently, this method is practical for 3D problems including complex crack patterns; however, using it may lead to spurious crack patterns [35,36].

The second group comprises intrinsic enrichment techniques where generally no additional unknowns are introduced to the approximation. This feature makes these techniques look more appealing. However, the well-known intrinsic enrichment in MMs called fully enriched basis [37,38] leads to a heavy computational cost caused by inverting a 7×7 matrix at each calculation point in order to compute shape functions and their associated derivatives. Calculating the stiffness matrix with a broader band-width is another consequence of employing the fully enriched basis technique. Nevertheless, this technique is not applicable to MLSRKM, in view of the fact that this method can just reproduce polynomial basis exactly in a global sense [14,17]. Another intrinsic enrichment technique, which in essence is applicable to all kernel-based methods and we deal with in the present work, is using the enriched weight functions (EWFs) introduced by Duflot and Nguyen-Dang [39]. They proposed some criteria based upon which three EWFs are constructed and then added to the crack-tip. They have shown that results are improved compared to an ordinary approximation. However, there are some concerns about EWFs described as follows. No displacement and stress analyses around the crack-tip were performed to demonstrate that these three EWFs are sufficient to represent the near-tip displacement field and capture the singularity of the stress field in the vicinity of the crack-tip. More importantly, there was no numerical experiment to show the effect of EWFs on the convergence behavior of a mesh-free Galerkin method if they are used in a mesh-free approximation space. Furthermore, by assuming post-processing routine of evaluating *J*-integral [40], there were no results showing the prominent path-independency property of stress intensity factors (SIFs) stemming from the *J*-integral values when EWFs are employed. As a matter of fact, this property plays an important role in crack propagation problems in order to predict the crack growth path more accurately. Here, it is shown that EWFs are not truly representative of the known solution around the crack-tip. Hence, we introduce the fully enriched weight functions (FEWFs) technique. Also, it is discussed that by introducing EWFs into MLSRKM, some modifications related to EWFs and MLSRKM are required. For demonstration, we focus on analyzing some 2D problems to show the efficiency and accuracy of the proposed technique as well as addressing the above-mentioned issues. It should also be noted that the enriching or modifying weight functions approach has been recently used to introduce discontinuity into the mesh-free approximation [41]. The resulting method allows a more straightforward simulation of multiple cracks, crack branching, and crack propagation without using any of the existing discontinuity criteria, such as visibility and diffraction methods and without introducing any additional unknowns and equations, as in extrinsic partition of unity-based methods. Therefore, the proposed technique in this paper together with the method in [41] can be considered as a powerful tool to solve the LEFM problems, even in 3D cases. Extension of this approach to analyze nonlinear fracture mechanics problems such as the Hutchinson–Rice–Rosengren (HRR) singular field model [42,43] is the subject of a future work.

The current paper is presented as follows: In Section 2, MLSRK approximation and the associated shape functions are derived. In Section 3, MLSRK approximation is introduced into the weak form, obtained from the Galerkin method, to achieve its discrete version, and some notifications on imposing essential boundary conditions (EBCs), numerical integration and the choice of the elements of MLSRK shape functions are given. In Section 4, it is discussed how to model a discontinuity in MMs. In Section 5, a brief overview on the enrichment techniques and their associated features is presented. In Section 6, FEWFs technique is introduced. How to extract SIFs is described in Section 7. Some numerical examples are presented in Section 8 to depict the performance of the proposed technique. Eventually, conclusions are drawn in Section 9.

2. Moving least square reproducing kernel approximation

The procedure of deriving shape functions or approximants associated with MLSRKM actually lies in its name. In fact, this procedure consists of two crucial steps: first, constructing a kernel function equipped to the least square technique and giving a reproduction or projection of a function, and second, using the moving process to achieve a global approximation throughout the domain without any need for meshing. As discussed in [14,15], the moving least square reproducing kernel integral takes the following form:

$$\mathscr{R}_{\varrho}^{m}u(\mathbf{x}) := \int_{\Omega_y} \mathbf{u}(\mathbf{y})\mathcal{K}_{\varrho}(\mathbf{y}-\mathbf{x},\mathbf{x})d\Omega_y \tag{1}$$

In relation (1), $\mathscr{R}_{\varrho}^{m}$ is the reproducing operator giving the reproduction or projection of $\mathbf{u}(\mathbf{x})$ – a sufficiently smooth function defined on a simply connected open set $\Omega\in\mathbb{R}^n$ – by the resolution $\boldsymbol{\varrho}$ with the highest polynomial order m which is employed to generate the polynomial vector $\mathbf{P}(\mathbf{x})$. $\mathbf{P}(\mathbf{x})$ is a complete m-order, l-component polynomial vector with $P_1(\mathbf{x})=1$. $\boldsymbol{\varrho}$ is the dilation parameter vector [14]; and, $\mathcal{K}_{\varrho}(\mathbf{y}-\mathbf{x},\mathbf{x})$ is defined by

$$\mathcal{K}_{\varrho}(\mathbf{y}-\mathbf{x},\mathbf{x}) := \mathcal{C}(\boldsymbol{\varrho},\mathbf{y}-\mathbf{x},\mathbf{x})\Phi_{\varrho}(\mathbf{y}-\mathbf{x}), \tag{2}$$

which represents the moving least square reproducing kernel formula. In (2), Φ is a weight function defined by

$$\Phi_{\varrho}(\mathbf{y}-\mathbf{x}) := \frac{C_n}{\boldsymbol{\varrho}^n}\Phi\left(\frac{\mathbf{y}-\mathbf{x}}{\boldsymbol{\varrho}}\right)\begin{cases} >0, & \mathbf{x}\in\operatorname{supp}\{\Phi_{\varrho}(\mathbf{y}-\mathbf{x})\}, \\ =0, & \text{otherwise}, \end{cases} \tag{3}$$