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Cover refinement of numerical manifold method for crack propagation simulation

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ABSTRACT

A cover refinement method is proposed for the numerical manifold method (NMM) to simulate crack propagation in brittle materials. New mathematical covers are defined for manifold elements near a crack tip. The refinement is done for corresponding mathematical covers of the selected manifold elements. The updating process of mathematical cover with respect to different boundary conditions is introduced in detail. When a mathematical cover is updated, the corresponding physical covers and manifold elements are updated accordingly. Furthermore, the loops of the considered domain are updated as well. Three numerical examples are analyzed to validate the proposed cover refinement method. The numerical results are all in good agreement with those results in the existing studies. It is demonstrated that the proposed cover refinement method has higher accuracy for crack propagation simulation comparing to the traditional numerical manifold method which has a consistent mathematical cover system. The proposed cover refinement method also does not significantly change the manifold elements at the vicinity of the crack tips. A rock slope model with a bilinear failure mode is simulated and progressive failure process of the rock slope is obtained, which demonstrates the applicability of the proposed method in practical rock engineering.

1. Introduction

Natural geological and engineering materials such as rock and concrete contain various inherent voids and cracks. When the external load exceeds a certain level, the existing cracks evolve in the material, and finally coalesce with each other. There are many challenges when numerical methods are used to simulate crack propagations in materials. Amongst, the simulation of moving crack tips is critical to derive a correct failure pattern and it has attracted great attention of researchers in the past decades. Various numerical techniques have been put forward to overcome the singularity of the stress field at the crack tip and great successes have been achieved.

The earlier studies focused on the local remeshing around a propagating crack tip, such as the automatic remeshing scheme [1,2], the new hybrid algorithm [3] and the moving mesh technique [4]. The minimal remeshing finite element method [5] was presented for crack growth with discontinuous enriched functions. The remeshing was needed only for severely curved cracks and places away from the crack tip. To deal with multiple boundaries and multiple materials, a solution based on an advanced remeshing and nodal relaxation technique [6] was proposed, and then three different crack growth criteria and the respective crack paths prediction for several test cases were compared [7]. A general study of the stability of variable-mesh dynamic calculations using an energy approach with remeshing was presented to establish the conditions which were necessary to ensure stability and allow control of energy transfers during the evolution of the mesh [8].

Other mesh refinement methods and subdomain or substructed methods have also achieved accurate results in crack propagation simulations. The effect of using collapsed quarter-point elements in conjunction with the adaptive refinement procedure in solving crack problems was investigated [9] and it was found that the efficiency of the adaptive procedure could be increased considerably. Furthermore, an automated adaptive remeshing procedure was presented [10] to simulate arbitrary shape crack growth in a 2D finite element mesh. Some schemes that split the domain into several subdomains or using radial basis functions to fracture problems were further investigated [11,12].

Recently, new numerical methods, such as the extended element method (XFEM) [13] and the numerical manifold method (NMM) [14], combined with the level set technique [15–17] and enriched function technique [13,18] achieved great success. An algorithm which couples the level set method with the extended finite element method to model crack growth was described [17]. In addition, a standard displacement-based enriched approximation

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method [19] was presented from the interaction of the crack geometry with the mesh. This technique allows the entire crack to be represented independently of the mesh, and so remeshing is avoided. The higher-order XFEM [20,21] was also used to modify the enriched element basis with the asymptotic crack tip displacement solutions as well as with the Heaviside function to improve the rate of convergence and enable the accurate approximation of solutions with jumps or kinks within elements. Furthermore, a new method [22] was presented for treating arbitrary discontinuities without additional unknowns, by an approximation space consisting of mesh-based, enriched moving least-squares functions near discontinuities and standard FE shape functions. Some important parameters controlling the accuracy of crack tip fields using the XFEM and statically admissible stress recovery, such as the order of quadrature, the number of retained terms in the crack tip asymptotic field, the number of enriched layers and use of arbitrary branch functions, a proper choice of the sampling points in the enriched element and the size of the domain of influence of moving least squares, were also investigated [23]. The singular enrichment surface with a cutoff function and the optimality of the coupling between the singular and the discontinuous enrichments were done in [24]. Nakasumi et al. [25] presented a method combined with mesh superposition technique and the XFEM to crack propagation analysis for large scale or complicated geometry structures. Some new Gaussian integration schemes for discontinuities and crack singularities in the XFEM [26] and some application in composite structures [18] as well as the enrichment of the XFEM by meshfree approximations [27] have also been studied. The detailed information can be found in the review paper [13,28]. In the meanwhile, crack propagation simulations with the NMM and Discontinuous Deformation Analysis (DDA) [29–33] have been described and the comparisons between the NMM and XFEM in crack propagation simulations have also been carried out. Similar to the XFEM, the NMM is able to simulate both continuum and discontinuum in a unified manner. It has the advantage to simulate heavily fractured solids using a regular or irregular cover system and it has been widely applied to simulate blocky rock mass deformation and stability under different static and dynamics loads. Different from the FEM and its extended versions which approximate the displacement field in a finite element using the node displacements, the NMM calculates the displacement fields in a manifold element using quantities at associated physical covers. Detail introduction of the NMM can be found in [14,30]. The mesh refinement of NMM predicting the crack propagation has ever been discussed by Tsay et al. [34] and Chiou et al. [35]. However, the introduction of the method, without considering the important points of refinement in the NMM, was not explained clearly and the examples illustrated mainly focused on problems with tensile loading.

In the present paper, a cover refinement method is proposed for the NMM to simulate crack propagation. New mathematical covers are defined for manifold elements near a crack tip. Not all frontal manifold elements at the vicinity of a crack tip but only those selected manifold elements satisfying certain conditions need to be refined. It is demonstrated that the cover refinement method has higher accuracy for crack propagation simulation comparing to the traditional numerical manifold method which has a consistent mathematical cover system throughout the simulation process. A few examples are used to validate the proposed cover refinement method. A rock slope model with a bilinear failure mode is simulated and a progressive failure of the slope is obtained, which demonstrated the applicability of the proposed method in practical rock engineering.

2. Fundamentals of the NMM

The NMM is based on three basic concepts, i.e. the mathematical cover, the physical cover and the manifold element. The mathematical cover system is a set of small patches that must large enough to cover the whole considered physical domain. Each small patch is termed as a mathematical cover, denoted by M_i, $i=1\sim n_M$. The physical covers are formed by both mathematical covers and physical domain. When a mathematical cover is divided by physical boundaries, only the fraction inside the physical domain forms a physical cover. If a mathematical cover M_i is completely divided into two or more smaller domains by joints or physical boundaries, these smaller separated domains are defined as physical covers, denoted by P_i^j, $j=1\sim n_P^i$. If a mathematical cover M_j has only one physical cover, and then the corresponding physical cover P_j^1 is simply denoted by P_j. The common domain of physical covers, e.g. P_i^{j1}, P_j^{j2} and P_k^{j3}, is defined as a manifold element, denoted by $E(P_i^{j1}, P_j^{j2}, P_k^{j3})$, where P_i^{j1}, P_j^{j2} and P_k^{j3} are the $j1$th, the $j2$th and the $j3$th physical cover of mathematical cover M_i, M_j and M_k, respectively.

In order to explain these three concepts clearly, an example is illustrated in Fig. 1. The regular hexagon with bold edges

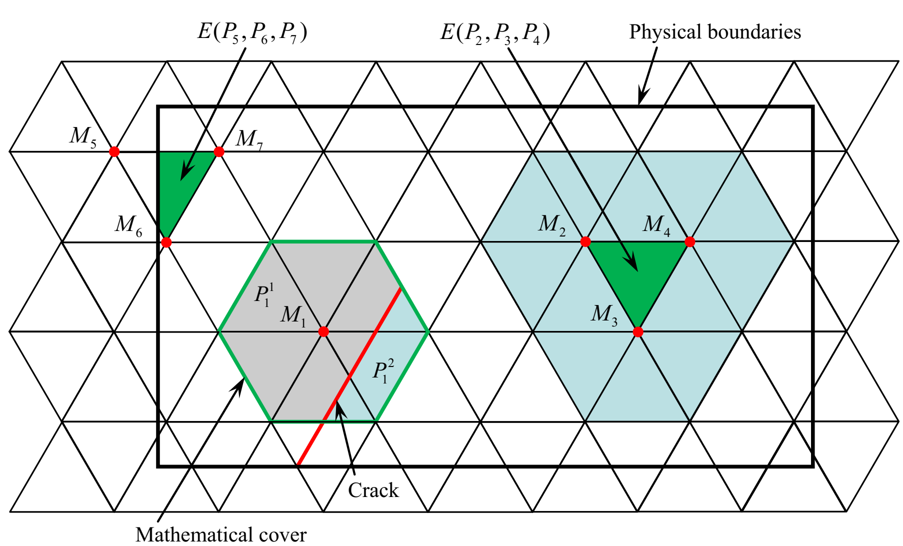

Fig. 1. Covers and elements in the NMM.