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Convolution quadrature methods for 3D EM wave scattering analysis

Amir Geranmayeh

Continental Automotive GmbH (I ID RD EE EL ED), VDO-Str. 1, 64832 Babenhausen, Germany

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The time-domain boundary integral equations describing the electromagnetic wave scattering from arbitrary three-dimensional metallic structures are solved by applying spectral domain finite-difference approximations while mapping from the Laplace domain to the z-transform domain. The validity of the results are verified through comparison with high-resolution finite difference time domain method results and the convergence rate of the introduced time-marching schemes is compared with the time basis functions expansion methods.

1. Introduction

The usage of boundary element methods to solve the time-domain integral equations (TDIE) [1] for the transient analysis of the wave radiation and scattering problems [2] has been of continuous interest in computational electromagnetic and numerical modeling communities for almost half a century [3]. The temporal discretization of the (TDIE) is commonly accomplished by either the implicit marching-on-in-time (MOT) schemes using subdomain Lagrange polynomial interpolation [4] or the always-stable marching-on-in-order/degrees (MOD) of Laguerre entire-domain bases [5]. An alternative approach for discretizing the time convolution integrals in the TDIE, competitive to the time basis functions expansions in the MOT or MOD recipes, is the Lubich's convolution quadrature methods (CQM) [6], using the (first or) second order backward finite difference (BFD) approximations in the Laplace domain. As a great advantage, in modeling dispersive dielectric material where the relative permittivity $\epsilon_r(s)$ and permeability $\mu_r(s)$ and the Green's functions are functions of s (e.g., in Debye equation for the complex permittivity), the usage of CQM for time-discretization in the Laplace domain let the frequency-dependent characteristics be directly incorporated into the time-domain solver [7].

The underlying physics describing the wave scattering process is time invariant [8], as the material properties do not change over time. The CQM, hence, can be utilized to transform continuous-time representation of the time-invariant integral kernel (system transfer function) to discrete-time domain while approximating the TDIE derivatives in the spectral domain. The CQM have been successfully applied to transient heat conduction [9], acoustic wave propagation [10], elastic and viscoelastic dynamic problems [11] as well as poroelastic formulations [12]. The CQM are called finite difference delay modeling (FDDM) when the scattering analysis of arbitrarily shaped three-dimensional (3D) structures is carried out in a marching style [13]. The FDDM is a provably stable method when the Lubich's CQM for the time discretization is used in conjunction with the Galerkin moment method for the spatial discretization [14]. In the mathematical literature, the method is called convolution quadrature when it applies to the single layer potential for the Helmholtz operator [15]. It is also called Tustin's method in digital signal processing and control theory to transform continuous-time representation (transfer function) of a linear time-invariant system to discrete-time domain.

In the CQM or FDDM methods, a conformal mapping from the Laplace domain to the z-transform domain (bilinear transform) based on a finite difference formula accomplishes the discretization in the z-domain, and the result is inverse transformed to create a time-domain method. The bilinear transformation preserves the stability by exact mapping every point of the $j\omega$-axis in the s-plane onto the unit circle $|z| = 1$ in the z-plane. Following this approach, each frequency response of the continuous-time system can be processed using a discrete-time filtering technique. The numerical solution of retarded functional equations can benefit from this frequency wrapping once the system's unit delays are replaced by first order all-pass filters z^{-1} in the discrete domain, as schematically depicted in Fig. 1.

The system eigenvalue analysis of the FDDM on small-scale case studies reported in [16] revealed that the CQM render a symplectic energy-conserving time integrator for the numerical solution of the TDIE. The temporal discretization is carried out by

E-mail address: amir.geranmayeh@continental-corporation.com

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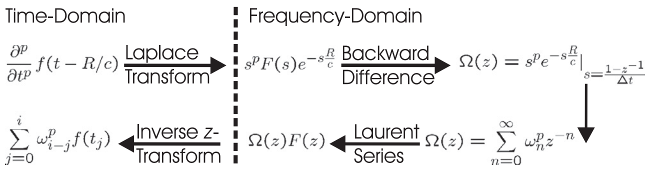

Fig. 1. Block diagram of the time discretization/integration procedures by the CQM or FDDM methods.

either first or second order BFD approximation when the transformed TDIE is mapped from the Laplace domain to the z-transform domain. Implicit Runge–Kutta schemes can also be applied for the temporal discretization when mapping from the Laplace domain to z-domain. Thereafter, s parameter is replaced with a matrix function of z and the inverse z-transform is computed numerically using the discrete Fourier transform (DFT) [17]. The BFD approximations of greater than second order are, however, never absolutely stable. The absolutely stable (A-stable) Radau IIA methods with two- and three-stage has third- and fifth-order convergence, respectively.

Wang et al. [13] determine the time derivative of the combined field integral equation (CFIE) in conjunction with the higher-order spatial bases. The time derivative of the integral equations, however, seeks for derivative of the transient excitation that may not be available for impulsive pulses. The CQM can also be applied to solve the original form of the electric field integral equation (EFIE) containing a time integral [16]. In this paper, the FDDM is also adopted to the primary EFIE without additional time derivative as well as separately to the magnetic field integral equation (MFIE) with a single time derivative, using the linearly varying divergence-conforming space basis functions. Additionally, the calculations of recursive convolutions are accelerated here using the time-FFT algorithms on Toeplitz block aggregates of the retarded interaction matrices [14].

2. TDIE and CQM methods

Let S denotes the surface of a perfect electric conducting (PEC) body that is excited by a transient electromagnetic field $\mathbf{E}^i(\mathbf{r},t)$. The total tangential electric field on S remains zero for all times. As a result, the induced surface current vector $\mathbf{J}(\mathbf{r},t)$ satisfies the following time-domain EFIE:

$$\frac{\mu}{4\pi}\frac{\partial}{\partial t}\int_S \frac{\mathbf{J}(\mathbf{r}',\tau)}{R}\,dS' - \frac{\nabla_\mathbf{r}}{4\pi\epsilon}\int_S\int_{-\infty}^{\tau}\frac{\nabla_{\mathbf{r}'}.\mathbf{J}(\mathbf{r}',t')}{R}\,dt'\,dS' = \underline{\mathbf{E}}^i(\mathbf{r},t) \tag{1}$$

where $\underline{\mathbf{E}}^i(\mathbf{r},t) = \hat{\mathbf{n}}\times(\hat{\mathbf{n}}\times\mathbf{E}^i(\mathbf{r},t))$, $R=|\mathbf{r}-\mathbf{r}'|$ and the observation point $\mathbf{r}$ and the source point $\mathbf{r}'$ indicate arbitrarily located points on the surface S and $\hat{\mathbf{n}}$ denotes an outward-directed unit vector normal to S at field point $\mathbf{r}$. The variable $\tau = t - R/c$ is the retarded time, in which the speed of wave propagation $c = 1/\sqrt{\mu\epsilon}$ is determined through the permeability μ and permittivity ϵ of the surrounding environment.

The derivative counterpart of the EFIE (DEFIE) is also of interest to preferably avoid the laborious computation of charge accumulation in solving the original EFIE (1) by the MOT methods. The DEFIE is obtained by taking a time derivative from both sides of (1)

$$\frac{\mu}{4\pi}\frac{\partial^2}{\partial t^2}\int_S \frac{\mathbf{J}(\mathbf{r}',\tau)}{R}\,dS' - \frac{\nabla_\mathbf{r}}{4\pi\epsilon}\int_S \frac{\nabla_{\mathbf{r}'}.\mathbf{J}(\mathbf{r}',t)}{R}\,dS' = \frac{\partial\underline{\mathbf{E}}^i(\mathbf{r},t)}{\partial t}. \tag{2}$$

Note that the Hertz vector potential can also be adapted to solve the EFIE [5] instead of direct solving for the unknown surface current. Evidently, the Hertz approaches demand extra post-processing stages for computation of some desired electromagnetic quantities. Excluding the temporal derivation on the excitation term, the Hertz approach results in formulations identical to the DEFIE (2).

Considering S as a closed surface, one may also consider the time-domain MFIE

$$\frac{\mathbf{J}(\mathbf{r},t)}{2} - \hat{\mathbf{n}}\times\frac{1}{4\pi}\int_{S_0}\left[\frac{1}{c}\frac{\partial\mathbf{J}(\mathbf{r}',\tau)}{\partial t}\times\frac{\mathbf{R}}{R^2} + \mathbf{J}(\mathbf{r}',\tau)\times\frac{\mathbf{R}}{R^3}\right]dS' = \hat{\mathbf{n}}\times\mathbf{H}^i(\mathbf{r},t) \tag{3}$$

where S_0 denotes the surface S from which the contribution of the singularity at $R=0$ has been removed [14].

To numerically solve the time-domain EFIE (1), DEFIE (2), and MFIE (3) or any linear combinations of them, they are first transferred to the Laplace domain

$$\frac{\mu s}{4\pi}\int_S \tilde{\mathbf{J}}(\mathbf{r}',s)\frac{e^{-sR/c}}{R}\,dS' - \frac{\nabla_\mathbf{r}}{4\pi\epsilon s}\int_S \nabla_{\mathbf{r}'}.\tilde{\mathbf{J}}(\mathbf{r}',s)\frac{e^{-sR/c}}{R}\,dS'\Big|_{\tan} = \underline{\tilde{\mathbf{E}}}^i(\mathbf{r},s) \tag{4}$$

$$\frac{\mu s^2}{4\pi}\int_S \tilde{\mathbf{J}}(\mathbf{r}',s)\frac{e^{-sR/c}}{R}\,dS' - \frac{\nabla_\mathbf{r}}{4\pi\epsilon}\int_S \nabla_{\mathbf{r}'}.\tilde{\mathbf{J}}(\mathbf{r}',s)\frac{e^{-sR/c}}{R}\,dS'\Big|_{\tan} = s\underline{\tilde{\mathbf{E}}}^i(\mathbf{r},s) \tag{5}$$

$$\frac{\tilde{\mathbf{J}}(\mathbf{r},s)}{2} - \frac{1}{4\pi}\hat{\mathbf{n}}\times\int_{S_0}\left[\frac{s}{c}\tilde{\mathbf{J}}(\mathbf{r}',s)\times\frac{\mathbf{R}}{R^2} + \tilde{\mathbf{J}}(\mathbf{r}',s)\times\frac{\mathbf{R}}{R^3}\right]e^{-sR/c}\,dS' = \hat{\mathbf{n}}\times\tilde{\mathbf{H}}^i(\mathbf{r},s). \tag{6}$$

The unknown induced surface current density $\tilde{\mathbf{J}}(\mathbf{r},s)$ is approximately expanded using Bernoulli's separation of variables in the space and Laplace domains by

$$\tilde{\mathbf{J}}(\mathbf{r},s) = \sum_{k=1}^{M}\tilde{I}_k(s)\mathbf{f}_k(\mathbf{r}). \tag{7}$$

where $\tilde{I}_k(s)$ are unknown weighting coefficients of the spatial vector basis functions $\mathbf{f}_k(\mathbf{r})$. Substituting (7) in the EFIE (4), the DEFIE (5), and the MFIE (6) respectively gives

$$\frac{\mu s}{4\pi}\sum_{k=1}^{M}\tilde{I}_k(s)\int_S\frac{\mathbf{f}_k(\mathbf{r}')}{R}e^{-sR/c}\,dS' - \frac{\nabla_\mathbf{r}}{4\pi\epsilon s}\sum_{k=1}^{M}\tilde{I}_k(s)\int_S\frac{\nabla_{\mathbf{r}'}\cdot\mathbf{f}_k(\mathbf{r}')}{R}e^{-sR/c}\,dS' = \underline{\tilde{\mathbf{E}}}^i(\mathbf{r},s) \tag{8}$$

$$\frac{\mu s^2}{4\pi}\sum_{k=1}^{M}\tilde{I}_k(s)\int_S\frac{\mathbf{f}_k(\mathbf{r}')}{R}e^{-sR/c}\,dS' - \frac{\nabla_\mathbf{r}}{4\pi\epsilon}\sum_{k=1}^{M}\tilde{I}_k(s)\int_S\frac{\nabla_{\mathbf{r}'}\cdot\mathbf{f}_k(\mathbf{r}')}{R}e^{-sR/c}\,dS' = s\underline{\tilde{\mathbf{E}}}^i(\mathbf{r},s) \tag{9}$$

$$\frac{1}{2}\sum_{k=1}^{M}\tilde{I}_k(s)\mathbf{f}_k(\mathbf{r}') - \frac{1}{4\pi}\hat{\mathbf{n}}\times\left[\sum_{k=1}^{M}\frac{s}{c}\tilde{I}_k(s)\int_S\frac{\mathbf{f}_k(\mathbf{r}')\times\mathbf{R}}{R^2}e^{-sR/c}\,dS' + \sum_{k=1}^{M}\tilde{I}_k(s)\int_S\frac{\mathbf{f}_k(\mathbf{r}')\times\mathbf{R}}{R^3}e^{-sR/c}\,dS'\right] = \hat{\mathbf{n}}\times\tilde{\mathbf{H}}^i(\mathbf{r},s). \tag{10}$$

Performing the Galerkin's testing procedure in space using the same set of vector basis functions $\mathbf{f}_m(\mathbf{r}), m=1,2,\ldots,M$, respectively, then gives

$$\frac{\mu s}{4\pi}\sum_{k=1}^{M}\tilde{I}_k(s)\int_S\mathbf{f}_m(\mathbf{r})\cdot\int_S\frac{\mathbf{f}_k(\mathbf{r}')}{R}e^{-sR/c}\,dS'\,dS + \frac{1}{4\pi\epsilon s}\sum_{k=1}^{M}\tilde{I}_k(s)\int_S\nabla_\mathbf{r}\cdot\mathbf{f}_m(\mathbf{r})\int_S\frac{\nabla_{\mathbf{r}'}\cdot\mathbf{f}_k(\mathbf{r}')}{R}e^{-sR/c}\,dS'\,dS = \int_S\mathbf{f}_m(\mathbf{r})\cdot\underline{\tilde{\mathbf{E}}}^i(\mathbf{r},s)\,dS \tag{11}$$