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Boundary element simulation of fatigue crack growth in multi-site damage

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ABSTRACT

This paper presents an efficient and automatic scheme for modelling the growth of multiple cracks through a two-dimensional domain under fatigue loading based on linear elastic fracture mechanics. The dual boundary element method is applied to perform an analysis of the cracked domain and the *J*-integral technique is used to compute stress intensity factors. Incremental crack propagation directions are evaluated using the maximum principal stress criterion and a combined predictor–corrector algorithm implemented for propagation angle and increment length. Criteria are presented to control the mesh used on the slower growing cracks in the domain, improving computational efficiency and accuracy by the use of virtual crack tips to avoid the need for severe mesh grading. Results are presented for several geometries with multi-site damage, and sensitivity to incremental crack length is investigated. The scheme demonstrates considerable advantages over the finite element method for this application due to simplicity of meshing, and over other boundary element formulations for modelling domains with large ranges of crack growth rates.

1. Introduction

Fatigue is a common mode of failure in materials subjected to cyclical loading. Here, cracks propagating through a structure reach a critical length, upon which a sudden and often catastrophic fracture failure occurs. The growth of cracks can be difficult to detect and monitor, necessitating the requirement for methods to simulate the behaviour of such cracks.

Modern structures and components can contain many thousands of sub-critical cracks and therefore an important design consideration is the extent of crack growth that can be sustained by the structure safely within its designed life. The damage tolerance approach to the fatigue assessment of a structure requires engineers to monitor cracks and also be able to calculate the remaining life. In order to make accurate calculations, detailed crack growth calculations must be performed. This is often done with the use of numerical elasticity calculations (e.g. with the finite element method (FEM)) in combination with a crack growth law.

A further consideration is multi-site damage, where a component contains multiple cracks of different sizes that can propagate at different rates and the interaction of cracks becomes significant. This phenomenon was brought into focus in 1988 when Aloha Airlines flight 243 experienced an explosive decompression following a structural failure of the fuselage in flight. The National Transportation Safety Board [1] determined in its accident report that the cause of the damage was failure of a fuselage lap joint from multi-site fatigue cracking. The consideration of multi-site damage has since become a critical consideration in aircraft design and maintenance; however, numerical simulation of multi-site fatigue crack growth remains challenging.

The increasingly complex geometries and crack interactions that are present in modern engineering structures require the development and use of numerical methods to simulate the propagation of cracks and to compute the resultant effect on stress fields. Linear Elastic Fracture Mechanics (LEFM) has long been used in damage tolerance assessments for cracked bodies. Here it is assumed that the crack tip plastic zone is small in comparison with the crack length. A complication arising in LEFM, and a particularly important one in performing numerical simulations, is that a stress singularity is present at the crack tip, so that values of local stress components become of limited use in assessing the stability of the crack and its propagation properties. Instead, stress intensity factors, K_I and K_{II}, are used to provide a convenient measure of crack stability, and also describe components, σ_{ij}, of the stress tensor in the vicinity of the crack tip. For a pure mode I crack, for example, the stress field around the crack tip is given, in the usual polar coordinate system (r, θ) centered on the crack tip, by

$$\sigma_{ij}(r,\theta) = \frac{K_I}{\sqrt{2\pi r}} f_{ij}(\theta) + \text{Higher order terms}\ldots \tag{1}$$

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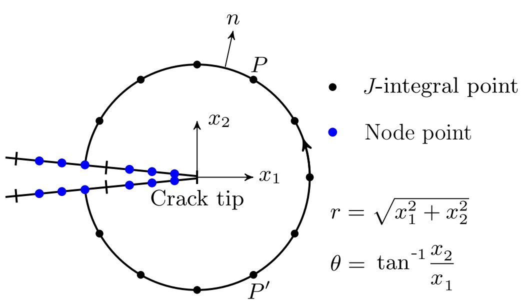

Fig. 1. Circular J-integral contour path and coordinate reference system relative to crack tip.

where K_I is the mode I stress intensity factor and f_{ij} is a dimensionless function of θ, the angular coordinate measured from the crack axis (see Fig. 1). As $r \to 0$, a singularity of order $r^{-1/2}$ develops and the stress $\sigma_{ij} \to \infty$. The stress intensity factor is thus a scaling factor which indicates the severity of the singular crack tip stress field.

In numerical simulations using any method based on LEFM, cracks are modelled in incremental analyses whereby a crack is incremented by a pre-calculated length and an analysis of the stress field in the structure is conducted and new stress intensity factors found. The crack propagation angle for each increment can be determined as a function of the stress intensity factors.

The FEM has been at the forefront of numerical simulation for decades and has been successfully used in fracture mechanics applications considering crack propagation [2]. However, when applied to iterative crack growth schemes, the FEM requires extremely fine meshing around crack tips to resolve the high stress gradients, and each increment in the analysis requires a remeshing of the domain. These problems were addressed by Moës et al. [3] who developed the extended finite element method (XFEM), which adds local enrichment to model the crack independently of the finite element mesh.

The scaled boundary finite element method [4] is an alternative approach which has benefits arising from (i) a reduced dimensionality of modelling, and (ii) a semi-analytical approach that can capture the leading order terms in the expansion (1) and yield stress intensity factors directly. However, the approach can become cumbersome for all but the simplest geometries.

The boundary element method (BEM) has become popular for modelling of fractures, and particularly for crack growth. This removes the need for remeshing, as only the boundary (including crack surfaces) are meshed, allowing subsequent increments to be considered simply by adding incremental elements to the existing mesh. The method is also well known for providing accurate boundary solutions to discontinuous fields. Using the classical form of the BEM, however, it is not possible to achieve a solvable system of equations in a single region, since collocation at coincident nodes on opposing crack surfaces causes the number of independent equations in the system to become insufficient. The Dual Boundary Element Method (DBEM) (see Hong and Chen [5], Portela et al. [6], and Chen and Hong [7]) addresses this by defining separate equations on collocated surfaces of the crack, one as a function of displacement and the other of traction, giving a non-singular system of equations which can be solved. The method has been applied to both single and multiple crack problems [6,8–11].

This paper presents an algorithm for the application of the DBEM to the incremental analysis of multi-site damage crack growth problems. In the evaluation of crack tip stress intensity factors, the J-integral is used, and for calculating crack paths the predictor–corrector technique proposed by Portela et al. [6] has been incorporated into a multiple crack algorithm. Fatigue analysis based on Paris law is adopted for simplicity, although other crack growth laws could easily be substituted. Further correction for increment length is implemented based on Salgado and Aliabadi [9], and new growth criteria are developed to control the extensions of cracks where a range of growth rates are present. Examples for multi-site damage fatigue problems are presented including single crack examples for validation and a series of multi-site damage applications with relevance to modern engineering structures.

The paper is organised in the following fashion. In Section 2, we present the DBEM and its application for fatigue crack growth. In Section 3, we extend the discussion to multi-site damage, presenting a new algorithm which includes the possibility of virtual crack extensions, and the approach is formalised in Section 4. In Section 5 we present validating examples, and discuss the performance of the algorithm. We close with some concluding remarks in Section 6.

2. The dual BEM for fatigue crack growth

We start by defining our two-dimensional domain $\Omega \subset \mathbb{R}^2$, having Lipschitz boundary $\Gamma := \partial\Omega$. The static equilibrium state of an elastic body described by the domain can be expressed in the following way:

$$\sigma_{ij,j} + b_i = 0 \tag{2}$$

where σ is the Cauchy stress tensor and b represents the body forces per unit volume. The stress tensor is related to components of the strain tensor, ε, through the constitutive relations:

$$\sigma_{ij} = C_{ijkl}\varepsilon_{kl} = \lambda\varepsilon_{kk}\delta_{ij} + 2\mu\varepsilon_{ij} \tag{3}$$

and the strains are displacement derivatives according to the definition:

$$\varepsilon_{ij} = \tfrac{1}{2}(u_{i,j} + u_{j,i}). \tag{4}$$

In the above λ is the Lamé constant, given by

$$\lambda = \frac{2\mu\nu}{1-2\nu}, \tag{5}$$

μ is the shear modulus, given by

$$\mu = \frac{E}{2(1+\nu)}, \tag{6}$$

δ is the Kronecker delta, E is Young's modulus, ν denotes Poisson's ratio and u_i are the displacements. In addition, the strains have to satisfy the compatibility equations:

$$\frac{\partial \varepsilon_{ij}}{\partial x_j \partial x_k} - \frac{\partial}{\partial x_i}\left(-\frac{\partial \varepsilon_{jk}}{\partial x_i} + \frac{\partial \varepsilon_{ik}}{\partial x_j} + \frac{\partial \varepsilon_{ij}}{\partial x_k} \right) = 0. \tag{7}$$

We consider a boundary value problem in which the above differential equations are solved subject to some Dirichlet and Neumann boundary conditions, respectively

$$u_i(\mathbf{x}) = \overline{u}, \quad \mathbf{x} \in \Gamma_u \tag{8}$$

$$t_i(\mathbf{x}) = \overline{t}, \quad \mathbf{x} \in \Gamma_t \tag{9}$$

where t_i is a traction component and the overbars denote prescribed values. The Dirichlet and Neumann boundaries, Γ_u and Γ_t, form the entire boundary, so $\Gamma = \Gamma_u \cup \Gamma_t$. Betti's reciprocal work theorem is applied to form the Boundary Integral Equation (BIE) for displacements at a point $\mathbf{x}' \in \Omega \backslash \Gamma$, in terms of tractions and displacements at all points on Γ. Taking for simplicity $b_i = 0$, $i = 1, 2$, this yields

$$u_i(\mathbf{x}') + \int_\Gamma T_{ij}(\mathbf{x}', \mathbf{x}) u_j(\mathbf{x}) \, \mathrm{d}\Gamma(\mathbf{x}) = \int_\Gamma U_{ij}(\mathbf{x}', \mathbf{x}) t_j(\mathbf{x}) \, \mathrm{d}\Gamma(\mathbf{x}), \tag{10}$$