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Unifiable multi-commodity kinematic wave model

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Abstract

In the literature, many kinematic wave models have been proposed for multi-class vehicles on multi-lane roads; however, there lacks an explicit model of unifiable multi-commodity traffic, in which different commodity flows can have different speeds and violate the first-in-first-out (FIFO) principle, but there exists a speed-density relation for the total traffic. In this study, we attempt to fill the gap by constructing and solving a unifiable multi-commodity kinematic wave model. We first construct commodity speed-density relations based on generic generating functions. Then for two commodities we discuss the properties of the unifiable kinematic wave model and analytically solve the Riemann problem with a combination of total and commodity kinematic waves. We propose a unifiable multi-commodity Cell Transmission Model (CTM) with a general junction model for numerical simulations of network traffic flows, which are unifiable but may violate the FIFO principle. We prove that the CTM is well-defined under an extended CFL (Courant et al., 1928) condition. With examples we verify the consistency between the analytical and numerical solutions and demonstrate the convergence of the CTM. We conclude with several follow-up research directions for unifiable multi-commodity kinematic wave models.

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Keywords: Multi-commodity kinematic wave model; First-in-first-out; Unifiable; Riemann problem; Total and commodity waves; Cell Transmission Model.

1. Introduction

Vehicles traveling on a road can be separated into different commodities based on their attributes, including their lanes, classes, aggressiveness, paths, and so on. Many kinematic wave models have been proposed to describe multi-commodity traffic dynamics in the evolution of both total and commodity densities, speeds, and flow-rates. For an M-commodity traffic, we can denote the density, speed, and flow-rate of commodity m ($m = 1, \cdots, M$) at x and t by $k_m(x,t)$, $v_m(x,t)$, and $q_m(x,t)$, respectively. We denote by $\vec{\mathbf{e}}_m$ the unit vector whose mth element is 1. The corresponding vectors of commodity densities, speeds, and flow-rates are denoted by $\vec{\mathbf{k}}(x,t) = \sum_{m=1}^{M} k_m(x,t)\vec{\mathbf{e}}_m$, $\vec{\mathbf{v}}(x,t) = \sum_{m=1}^{M} v_m(x,t)\vec{\mathbf{e}}_m$, and $\vec{\mathbf{q}}(x,t) = \sum_{m=1}^{M} q_m(x,t)\vec{\mathbf{e}}_m$, respectively. Correspondingly, the total traffic density, speed, and flow-rate are denoted by $k(x,t)$, $v(x,t)$, and $q(x,t)$, respectively. Hereafter (x,t) is omitted unless necessary.

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Multi-commodity kinematic wave models are defined based on the following rules:

(R1) Additive relations between commodity and total densities and flow-rates:

$$k = \sum_{m=1}^{M} k_m, \quad q = \sum_{m=1}^{M} q_m. \tag{1}$$

(R2) Commodity and total constitutive laws ($m = 1, \cdots, M$):

$$q_m = k_m v_m, \quad q = kv. \tag{2a}$$

(R3) Commodity speed-density relations ($m = 1, \cdots, M$):

$$v_m = \eta_m(\vec{\mathbf{k}}). \tag{2b}$$

(R4) Commodity flow conservation equation ($m = 1, \cdots, M$):

$$\frac{\partial k_m}{\partial t} + \frac{\partial q_m}{\partial x} = 0. \tag{2c}$$

(R5) Weak solutions: discontinuous shock waves can arise even from continuous initial and boundary conditions.
(R6) Entropy conditions: weak solutions should be unique and satisfy some physical laws (Lax, 1972; Ansorge, 1990).

From R2, R3, and R4, the kinematic wave model of M-commodity traffic can be written as a system of M conservation equations:

$$\frac{\partial k_m}{\partial t} + \frac{\partial k_m \eta_m(\vec{\mathbf{k}})}{\partial x} = 0, \tag{3}$$

whose solutions exist and are unique subject to R5 and R6. That is, with the six rules (R1)-(R6), (3) is well-defined, and one can calculate total and commodity variables at any time and location under proper initial and boundary conditions. Among the six rules, R1, R2, R4, and R5 apply to any multi-commodity traffic flows, but R3 and R6 are respectively related to static and dynamic characteristics of specific traffic systems. Therefore, different commodity speed-density relations and entropy conditions should be developed for different multi-commodity traffic systems.

From (1) and (2), the total traffic speed is also a function of all commodity densities:

$$v = \eta(\vec{\mathbf{k}}) \equiv \frac{\sum_{m=1}^{M} k_m \eta_m(\vec{\mathbf{k}})}{\sum_{m=1}^{M} k_m}. \tag{4}$$

In the following we define two important properties of multi-commodity traffic:

- First-in-first-out (FIFO). We call a multi-commodity traffic flow FIFO if and only if all commodities have the same speed at the same location and time:

$$v_1 = \cdots = v_M = v, \tag{5a}$$

which is equivalent to that all commodities have the same speed-density relation according to (4):

$$\eta_1(\vec{\mathbf{k}}) = \cdots = \eta_M(\vec{\mathbf{k}}) = \eta(\vec{\mathbf{k}}), \tag{5b}$$