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On the uniqueness of equilibrated dynamic traffic flow patterns in unidirectional networks

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Abstract

Uniqueness of the dynamic user-equilibrium assignment is still an important issue. This paper proves uniqueness with a milder condition compared to past studies and shows another counterexample to the uniqueness. A *unidirectional network*, in which any node on any shortest route has a unique node potential, is introduced. Orders of vehicles are determined by this node potential so that, given any two vehicles passing through the same node, the lower potential vehicle arrives at the node before the higher potential vehicle. It is shown that, for a unidirectional network in equilibrium, the link travel times and traffic volumes of congested links are uniquely determined. Moreover, a simple non-unidirectional network having multiple equilibria is introduced. This example exhibits the importance exhibits importance of unidirectional-network structure to prove uniqueness.

Keywords: dynamic traffic assignment, dynamic user equilibrium, unidirectional network, uniqueness, multiple equilibria

1. Introduction

Uniqueness of an equilibrium solution is still a main issue of the dynamic user-equilibrium (DUE) assignment problem. While uniqueness of static user-equilibrium assignment problems can be simply proven by an equivalent convex optimisation problem (Beckmann et al. (1956)), no such simple and comprehensive approach is known to prove uniqueness of the DUE assignment problem. Without a proof of uniqueness for DUE, we cannot guarantee that a solution solved by a certain methodology will be realised in the real world because there may be other solutions that are not found.

The issue of the uniqueness has not been very frequently investigated compared to other major topics of DUE problems, such as solution methods, and hence findings by past studies are rather limited (see Iryo (2013) for a detailed review for uniqueness and other important properties of DUE). It has been well known that showing the (strict) monotonicity of travel cost is a sufficient condition for uniqueness of a user-equilibrium solution (Smith (1979)). Several studies proposing solution methods have mentioned the monotonicity of travel cost or similar mathematical properties for a condition of the uniqueness (e.g. Lo and Szeto (2002), Huang and Lam (2002), Friesz and Mookherjee (2006),

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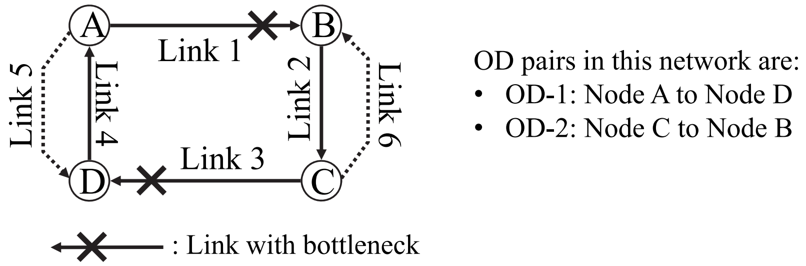

Fig. 1. Example of a network with loop

Perakis and Roels (2006), Friesz et al. (2011), Han et al. (2015)). However, no clear relationship between these mathematical conditions and characteristics of DUE problems has known except for the cases in which no route has two or more bottlenecks in the network. Such networks are called *single-bottleneck-per-route* networks. Smith and Ghali (1990), Mounce (2006), and Mounce (2007) mentioned this property. Mounce (2007) used this to prove that there exists only one convex equilibrium solution set in such a case when vehicles' departure times from their origins are fixed. The single-bottleneck-per-route property can also be used to prove that an equilibrium solution set is convex when drivers simultaneously select both departure times and routes (Iryo and Yoshii (2007)).

While the monotonicity of travel cost is mathematically useful to prove uniqueness, it is not a general property of general DUE problem. Indeed, it has been known that the monotonicity of travel cost does not hold in a simple network in which only one origin-destination pair exists (Kuwahara (1990), Mounce (2001), Mounce and Smith (2007)).

Proving uniqueness of DUE in general settings is actually impossible because it is known that the uniqueness is not a general property of DUE solutions. It has been shown that there exist cases in which non-unique equilibrated congestion patterns are found (Iryo (2011a), Iryo (2015)). This study uses a network including a *loop* in it. Fig. 1 is the test network proposed by Iryo (2011a), in which vehicles travelling from Node A to D first run on Link 1 and then Link 3, while those travelling from Node C to B run on Link 3 and then Link 1. Such a loop structure of the network allows vehicles traverse two links in the reverse-order, causing the existence of the multiple equilibria.

The consideration that the loop may interfere uniqueness of equilibrium gives rise to the idea of finding a condition for the uniqueness to hold, that is, *will the non-existence of loop structure guarantee uniqueness?* Actually, the single-bottleneck-per-route networks have this property. Hence, it should be worthwhile to investigate the uniqueness property assuming that there is no loop in a network.

A technique called *time-decomposition* should be useful to investigate uniqueness in a network with no loop. The time-decomposition technique was proposed by Kuwahara and Akamatsu (1993). Akamatsu (2000), Akamatsu (2001), Akamatsu and Heydecker (2003a), Akamatsu and Heydecker (2003b), Waller and Ziliaskopoulos (2006), Iryo (2010), and Iryo (2011b) used it or similar methods to develop a solution method or investigate solution properties. The technique assumes networks in which only a single origin or single destination exists, called *single-origin* or *single-destination networks*, then determines the order of vehicles so that vehicles with earlier orders enter every link earlier than vehicles with later orders. To prove uniqueness, Mounce and Smith (2007) implicitly incorporated this idea to show that the uniqueness of equilibrated congestion pattern is guaranteed by the bottleneck model if there exists only one origin-destination pair and vehicles' departure times are fixed. Akamatsu (2000), Akamatsu and Heydecker (2003b) mentioned the uniqueness of DUE when all links are congested or all congested links are specified before finding a solution in single-origin networks. However, as far as we know, these studies are only cases incorporating the concept of time-decomposition to prove the uniqueness of DUE.

The present paper aims to show a proof of uniqueness with a condition that is milder than those in past studies and give a counterexample to uniqueness to show that this condition is important to maintain uniqueness. A network structure called *unidirectional network* is introduced for the proof. In this study, the uniqueness is defined by uniqueness of link travel times, i.e. the congestion pattern is always identical between two equilibrium solutions, while uniqueness of a link traffic flow pattern is also investigated. In a unidirectional network, any nodes included within a shortest route of any OD pair has a unique node potential; the travel time of a shortest route between any two nodes is equal