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# The discrete-time second-best dynamic road pricing scheme

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**Abstract**

Congestion pricing is widely recognized as an efficient instrument for alleviating road congestion. Most of existing road pricing scheme are developed based on traffic equilibrium model. The equilibrium can be looked as a traffic state desired by traffic management authorities. However, when the traffic system has multiple equilibria if the initial traffic state falls beyond the attraction domain, it may not converge to the desired equilibrium under the pricing based on the traffic equilibrium through a day-to-day adjustment process (Bie and Lo, 2010). In this study, we aim to develop a more practical second-best dynamic road pricing scheme implemented on a subset of links, which can drive the traffic evolution towards the desired second-best traffic user equilibrium state from any initial traffic state. This second-best dynamic pricing scheme has the following characteristics: (i) the dynamic pricing is discrete-time scheme, as opposed to most of existing continuous dynamic pricing scheme; (ii) the derivation of the dynamic pricing is applicable to very general day-to-day traffic dynamic model; (iii) the dynamic pricing can direct the traffic system to converge to the desired equilibrium from any initial traffic state even multiple equilibria exist. This study also presents rigorous proofs and numerical tests to verify above these characteristics of our dynamic pricing scheme.


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*Keywords:* second-best road pricing; day-to-day traffic dynamic model; dynamic road pricing; multiple equilibria

## 1. Introduction

Roadway congestion is one of the most challenging problems faced by major cities, which has not only brought about enormous economic losses, but environmental pollution. These losses can be avoided in principle, because they mostly result from socially inefficient choices by individual drivers. It has been recognized that road pricing is an effective instrument for alleviating roadway congestion through implementing influence on the traffic demand and traffic flow assignment. The suitable tolls charged on certain links can make the limited road capacity efficiently utilized. The initial idea comes from Pigou (1924), who used an example of congestion road to show the externalities and optimal congestion charges. Walters (1961), Beckmann (1967) and Vickrey (1969) are also the representative and classical works on both intellectual and practical developments of road pricing.

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The first-best pricing or the marginal cost pricing is the theoretical basis of road pricing. This theory states that the user of road should pay toll to internalize the congestion externality or additional cost that the road user imposes on other users. By doing so, each road users will bear the marginal social cost of road use other than the marginal private costs (Yang and Huang, 2005), and the system optimum (SO) can be obtained. There are various extensions of the first-best pricing, e.g., multiclass, multicriteria traffic equilibrium, link flow interactions, and stochastic user equilibrium (Dafermos, 1973; Yang, 1999; Yang and Huang, 2004).

Although the first-best pricing scheme has perfect theoretical justification, it is impractical to levy road pricing on each network link in view of the operating cost and public acceptance. Therefore, the first-best pricing scheme is of little practical interests. As a result, various second-best pricing scheme have been proposed. For example, cordon based road pricing scheme, which is one type of second-best pricing scheme, has been put into practice for managing road congestion in congested urban areas in cities like Singapore, Trondheim and Bergen. Most researches in the context of the second best-pricing problems investigated the determination of toll levels for given charge locations. A classic version of this type of problem studies the two route problem, in which an untolled alternative road is available paralleling with the toll road. This problem was first studied by Levy-Lambert (1968) and Marchand (1968), and more recently by Braid (1996), Verhoef et al. (1996), Liu and McDonald (1998), De Palma and Lindsey (2000) and Verhoef and Small (1999). For a general analytical solution for the second-best problem where not all links of a congested transportation network can be tolled, interested readers may refer to Verhoef (2002), Yang and Huang (2005) and Lawphongpanich et al. (2006).

Most previous studies of the either first-best or second-best road congestion pricing employed approaches that are based on a stationary equilibrium traffic flow pattern in traffic networks to decide how much congestion pricing should be levied at the traffic equilibrium state. From the angle of traffic management, the traffic equilibrium state of first or second best road pricing scheme can be looked as a desired objective of traffic agencies, which should be achieved after certain time period if road pricing scheme is implemented. In fact, travelers adjust their traveling routes on a day-to-day basis and result in link or route flows evolution over time before reaching an equilibrium state. So, it is very natural to ask whether the traffic system can eventually reach the desired objective traffic equilibrium state under the fixed road pricing obtained by equilibrium model. As was pointed out by Horowitz (1984), even for a well-behaved system whose equilibrium solution was known to exist, depending on the dynamic route adjustment process, the system might still fail to converge to equilibrium. Even if a traffic system has already been staying at an equilibrium state, the traffic flow pattern would probably fall into a disequilibrium state because of the perturbation of controlled inputs, exogenous information, or random events, and then start to adjust again toward a new equilibrium (Guo et al., 2015a). Therefore, it is imperative to develop dynamic congestion management measures, which will direct the traffic system to reach and stay at certain equilibrium state desired by traffic authorities. Moreover, the advent of new technologies for the provision of information to automobile drivers and for the tracking of individual vehicles make it possible to implement dynamic congestion management. Dynamic pricing can be classified into two categories, i.e., day-to-day dynamic and within-day pricing according to the time scale of its application. In this study, we would focus on the day-to-day time scale dynamic pricing.

Compared to congestion pricing based on traffic equilibrium, the research works on day-to-day dynamic congestion pricing are much less. Sandholm (2002) applied evolutionary game approach to study the day-to-day dynamic congestion pricing which can force traffic system to reach system optimal state. In their study, the variable congestion pricing is based on marginal travel cost and the elastic and inelastic traffic demand are both considered. Friesz et al. (2004) proposed a disequilibrium day-to-day dynamic pricing scheme, which maximizes the net social benefit over the planning period, considering drivers' day-to-day behavior articulated in continuous time, revenue constraints and taking the form of ordinary differential equations. However, the proposed dynamic congestion pricing scheme using the continuous time optimal control theory does not guarantee that the traffic system will evolve to an objective state (e.g., SO) of traffic system that may be more desirable for traffic management agencies to achieve. Moreover, they made strong plausible regularity assumptions in their study; for example, the path cost function is assumed to be convex and monotonically increasing. Taking into account a more general behavior adjustment process and making less restrictive assumptions, Yang and W. Y. (2006) developed a day-to-day dynamic congestion pricing strategies that can force the traffic system to evolve from the status quo to SO. They showed that under, their dynamic pricing scheme, the steady state would be SO when the day-to-day dynamic traffic flow becomes stationary. Moreover, they claimed that the evolution to SO may be independent of the underlying day-to-day behavior adjustment process. More