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Minimal Parameter Formulations of the Dynamic User Equilibrium using Macroscopic Urban Models: Freeway vs City Streets Revisited

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Abstract

This paper investigates the dynamic user equilibrium (DUE) on a single origin-destination pair with two alternative routes, a freeway with a fixed capacity and the surrounding city-streets network, modeled with a network macroscopic fundamental diagram (NMFD). We find using suitable transformations that only a single network parameter is required to characterize the DUE solution, the freeway to NMFD capacity ratio. We also show that the stability and convergence properties of this system are captured by the constant demand case, which corresponds to an autonomous dynamical system that admits analytical solutions. This solution is characterized by two critical accumulation values that determine if the steady state is in free-flow or gridlock, depending on the initial accumulation. Additionally, we also propose a continuum approximation to account for the spatial evolution of congestion, by including variable trip length and variable NMFD coverage area in the model. It is found that gridlock cannot happen and that the steady-state solution is independent of surface network parameters. These parameters do affect the rate of convergence to the steady-state solution, but convergence rates appear virtually identical when time is expressed in units of the NMFD free-flow travel time.

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Keywords: dynamic traffic assignment, macroscopic fundamental diagram

1. Introduction

The simplest models that incorporate the effects of congestion on urban areas are reservoir models. They are based on vehicle conservation inside the reservoir with its outflow given by a known function of the accumulation. This function, called the Network Macroscopic Fundamental Diagram (NMFD), was first introduced in Godfrey (1969) and later used by Mahmassani et al. (1984, 1987), but only recently was shown to have strong empirical support suggesting a rather stable shape Daganzo (2007); Geroliminis and Daganzo (2007); Wang et al. (2015). Many control applications have been proposed since (e.g., Haddad and Geroliminis, 2012; Aboudolas and Geroliminis, 2013; Ampountolas et al., 2014; Hajiahmadi et al., 2013; Knoop and Hoogendoorn, 2014; Yildirimoglu et al., 2015; Kouvelas et al., 2016).

Apart from the shape of the NMFD, the main assumption of a reservoir model is that the travel time of a vehicle entering at time t is a function of the accumulation at the *same* time, which makes it applicable only when the inflow varies slowly. Otherwise it may be subject to the "infinite-wave-speed problem" due to the lack of a space dimension in the model. Thus, when fast inflow variations occur, they have immediate repercussions on the outflow meaning that information travel is infinitely fast inside the reservoir. Improved reservoir models have been proposed that guaranty

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that all vehicles travel their trip length before exiting (Arnott, 2013; Fosgerau, 2015; Lamotte and Geroliminis, 2016; Mariotte, 2016) at the expense of mathematical tractability. But when demand varies slowly, the simple reservoir gives good approximations, with the big advantage of being analytical. This has allowed extensions such as congestion pricing (Arnott, 2013; Daganzo and Lehe, 2015).

This paper focuses on the simple reservoir model in the context of dynamic traffic assignment (DTA), in particular the dynamic user equilibrium (DUE) when there is the alternative of reaching the destination using a freeway of limited capacity. Although there have been efforts in the past to combine DTA and MFD (Yildirimoglu and Geroliminis, 2014a; Leclercq and Geroliminis, 2013), existing methods are algorithmic or numerical. Apart from the numerical errors introduced, this approach makes it difficult to answer fundamental questions such as: (i) what are the parameters, or combinations thereof, that affect the solution? (ii) under what conditions is the simple reservoir model a good approximation? (iii) will the system converge to gridlock and how fast?

To answer these and other questions, here we establish the minimum set of parameters needed to fully characterize the DUE solution. Towards this end, section 2 considers the simplest problem of a single NMFD (no DUE) to show that the accumulation evolution is characterized by two critical accumulation values that determine if the steady state is in free-flow or gridlock, depending on the initial accumulation. It also shows that the stability and convergence properties of this system are captured by the constant demand case, which admits analytical solutions. Section 3 shows that only a single network parameter is required to characterize the DUE solution, the freeway to NMFD capacity ratio. Section 4 proposes a continuum approximation to account for the spatial evolution of congestion, by including variable trip length and variable NMFD coverage area, typically assumed constant in the literature (e.g., Haddad and Geroliminis, 2012; Aboudolas and Geroliminis, 2013; Hajiahmadi et al., 2013; Leclercq et al., 2015; Yildirimoglu and Geroliminis, 2014b). Finally section 5 presents a discussion.

2. Single NMFD loading

We start by considering the simplest problem of the dynamic loading of a surface network using a single NMFD; there is no freeware alternative, and therefore no DUE. It turns out that the solution to this problem is the building block for the following sections.

We assume that the surface network can be well described by an outflow-NMFD, $f(n)$, with capacity $\mu \equiv \max_n f(n)$. This function gives the production, i.e. the number of trip completions per unit time, as a function of the number of vehicles in the network, n. Let $\lambda(t)$ be the demand inflow into the network at time t. [1] The NMFD dynamics studied in this paper are given by the following ordinary differential equation (ODE):

$$\textbf{n-ODE:} \begin{cases} n'(t) = \lambda(t) - f(n), & \text{(reservoir dynamics)} \quad (1a) \\ n(0) = n_0, & \text{(initial conditions)} \quad (1b) \end{cases}$$

where primes denote differentiation, $\lambda(t)$ the inflow and n_0 specify the initial conditions. A key point is whether or not demand is restricted to the supply function of the NMFD, Ω; i.e.,

$$\lambda(t) \leq \Omega(n(t)). \qquad \text{(supply constraint)} \qquad (2)$$

Although this appears to be a standard assumption in the latest literature, we argue that this constraint negates the fact that distance traveled within the reservoir may increase with congestion. Therefore, we are interested here in the *unconstrained* system solution. We will show how solutions to the constrained system can be derived from the solutions presented here.

It is convenient to express system dynamics in terms of the dimensionless variables occupancy, $k(t)$, and demand intensity $\rho(t)$:

$$k(t) \equiv n(t)/(\kappa L), \qquad 0 \leq k(t) \leq 1, \qquad \text{(occupancy)} \qquad (3a)$$

$$\rho(t) \equiv \lambda(t)/\mu, \qquad \text{(demand intensity)} \qquad (3b)$$

[1] The word "demand" is used in the traffic flow context to distinguish the willingness to traverse a bottleneck (demand) from the ability to do so (flow); it is not used in the economics context to reflect cost elasticity. Here, the demand is an inelastic and exogenous time-dependent function.