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The morning commute in urban areas with heterogeneous trip lengths

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Abstract

This paper investigates the equilibrium properties of the morning commute problem at the network level with heterogeneous trip lengths. Congestion is modeled with a Macroscopic Fundamental Diagram relating the space-mean speed of a network to the vehicular accumulation. It is shown for a large class of scheduling preferences that if users have continuously distributed characteristics, the network accumulation at equilibrium is a continuous function of time. With $\alpha - \beta - \gamma$ preferences and under certain conditions, a partial First-In, First-Out (FIFO) pattern emerges at equilibrium among early and late users. This FIFO pattern is strict only within families of users having heterogeneous trip lengths and identical preferences, or vice versa. Simulation results confirmed that an attracting steady-state exists for a wide range of demands and that the predicted patterns are indeed observed.

Keywords: traffic congestion, departure time, trip length, scheduling preferences, equilibrium, pricing

1. Introduction

The morning commute has historically been studied with point bottlenecks, following the seminal paper of Vickrey (1969). Urban networks however, cannot be modeled as collections of independent point bottlenecks. In fact, congestion propagates from one bottleneck to its neighbors, creating connected components of congestion that grow and may extend to the whole network (Ji et al., 2014). In the face of this complexity, an attractive solution consists in changing the modeling scale, from a point bottleneck to the network level (Small and Chu, 2003; Geroliminis and Levinson, 2009). This approach relies on empirically supported relationships (see e.g. Geroliminis and Daganzo (2008) and Buisson and Ladier (2009)), referred to as Macroscopic Fundamental Diagrams (MFD), which describe the dynamics

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of congestion under some homogeneity conditions with just a few variables such as accumulation, space-mean flow, trip completion rate and speed.

The consequences of this change of scale are, however, not fully understood yet. The first works combining an MFD with departure time choice called attention to the cost of hypercongestion, i.e. the phenomenon by which vehicle flow decreases with accumulation when accumulation exceeds a critical level (Small and Chu, 2003). Geroliminis and Levinson (2009) and Fosgerau and Small (2013) argued that by maintaining the system at the flow-maximizing accumulation, the benefits of congestion pricing with homogeneous users could be even greater than with Vickrey's bottleneck, as the duration of the peak hour could be shortened. Arnott (2013) showed that further gains can be obtained by maintaining the accumulation always *below* its flow-maximizing value, at a level that increases with the peak duration. While they reach similar conclusions, the aforementioned papers utilized different assumptions to deal with complex dynamics involving endogenous delays. Small and Chu (2003) and Geroliminis and Levinson (2009) assumed that travel time is entirely determined by the conditions at a single instant (e.g. at the arrival), Fosgerau and Small (2013) considered a piece-wise constant decreasing branch for the MFD of flow versus accumulation and Arnott (2013) assumed that at any time, all users have the same probability of exiting the network, independently of the time at which they started their trip. Yet, it is only very recently that Fosgerau (2015) and Daganzo and Lehe (2015) recognized how trip length heterogeneity challenges the fundamental FIFO assumption and started investigating its impacts on the morning commute.

Surprisingly however, Fosgerau (2015) and Daganzo and Lehe (2015) reached very different conclusions on the role of trip length. Using mathematically convenient but unconventional exponential-type scheduling preferences, Fosgerau (2015) showed that under some assumptions, the user equilibrium exhibits the so-called regular sorting property, i.e. two users differing only by their trip length sort according to a Last-In, First-Out (LIFO) pattern where the user with the longest trip starts earlier and finishes later. On the other hand, Daganzo and Lehe (2015) proved for the more conventional $\alpha - \beta - \gamma$ scheduling preferences but with a less realistic congestion mechanism (actually very similar to a point bottleneck) that the social optimum exhibits the FIFO property. Based on these theoretical considerations, Daganzo and Lehe (2015) also proposed a usage-based toll maintaining the accumulation near its flow-maximizing value and demonstrated numerically its benefits on the same realistic congestion mechanism as in Fosgerau (2015). The question of the prevailing sorting pattern with heterogeneous trip lengths remains not fully solved, despite its crucial role in congestion management.

This paper investigates the morning commute problem with inelastic demand and an MFD relating speed to accumulation. Our focus is primarily on the impact of trip length heterogeneity, but we also study the impact of heterogeneity in the scheduling preferences. In line with most of the literature, we rely on the so-called "fluid approximation" (Newell, 1982), in which the stochastic dynamics of a large number of agents are modeled by a deterministic real-valued process. With Vickrey's constant capacity bottleneck, this assumption, together with some convexity assumptions on the schedule penalties, permitted to prove the existence and the uniqueness of an equilibrium distribution of arrival times (Smith, 1984; Daganzo, 1985). In this paper, we leave aside the existence and uniqueness questions as they would require tedious derivations beyond the scope of the paper (even for a bottleneck model with much simpler dynamics, the proofs in Smith (1984) and Daganzo (1985) contain a significant number of modeling assumptions, but also very long derivations). Instead, we analytically characterize an hypothetical equilibrium and confirm using simulations that (i) an attracting steady-state exists for a wide range of demands and that (ii) the patterns theoretically predicted are indeed observed.

The complexity of the problem is mainly due to an endogenous delay in the dynamical model (more information is provided in Section 2.1). This makes a fully analytical solution intractable, as also pointed out by Arnott (2013). Nevertheless, in this work we are able to mathematically prove important properties of the equilibrium solution and develop a numerical simulation of the detailed model that confirms some of the proofs and provides further insight. More specifically, this paper has three main contributions. First, we show that with a continuum of users having continuously distributed characteristics, accumulation and speed are continuous functions of time. This result is valid for a large class of scheduling preferences and contrasts with the discontinuities resulting from the assumption of homogeneous trip lengths (Arnott et al., 2016). Second, we assume $\alpha - \beta - \gamma$ preferences and, under certain conditions, demonstrate that they result in a partial FIFO pattern among early and late users at equilibrium. This FIFO pattern is strict only within families of users having identical $\alpha - \beta - \gamma$ preferences and heterogeneous trip lengths, or vice versa. Yet, simulation results indicate that it influences the overall properties of the equilibrium, and in particular