Available online at www.sciencedirect.com

ScienceDirect

Transportation Research Procedia 23 (2017) 624–644

Transportation Research Procedia

www.elsevier.com/locate/procedia

22nd International Symposium on Transportation and Traffic Theory

# An optimization modeling of coordinated traffic signal control based on the variational theory and its stochastic extension

Kentaro Wada [a,*], Kento Usui [b], Tsubasa Takigawa [c], Masao Kuwahara [d]

[a] *Institute of Industrial Science, The University of Tokyo, Tokyo, Japan*
[b] *Nagoya Railroad Co., Ltd, Nagoya, Aichi, Japan*
[c] *East Nippon Expressway Co., Ltd., Saku, Nagano, Japan*
[d] *Graduate School of Information Sciences, Tohoku University, Sendai, Miyagi, Japan*

**Abstract**

This study considers an optimal coordinated traffic signal control under both deterministic and stochastic demands. We first present a new mixed integer linear programming (MILP) for the deterministic signal optimization wherein traffic flow is modeled based on the variational theory and the constraints on a signal control pattern are linearly formulated. The resulting MILP has a clear network structure and requires fewer binary variables and constraints as compared with those in the existing formulations. We then extend the problem so as to treat the stochastic fluctuations in traffic demand. We here develop an accurate and efficient approximation method of expected delays and a solution method for the stochastic version of the signal optimization by exploiting the network structure of the problem. Using a set of proposed methods, we finally examine the optimal control parameters for deterministic and stochastic coordinated signal controls and discuss their characteristics.



*Keywords:* coordinated signal control; kinematic wave theory; variational theory; random arrival; Clark approximation; cross-entropy method

## 1. Introduction

A system of traffic signals plays a vital role in determining the performance of arterial roads and/or road networks. Considerable research has been devoted to designing signal control parameters, such as cycle lengths, green splits, and offsets for coordinating traffic signals; several simulation-based heuristic optimization methods (e.g., Park et al., 2000; Maher et al., 2013) and commercial softwares (e.g., TRANSYT) are available for determining control parameters considering detailed and realistic phenomena and constraints (see Papageorgiou et al., 2003; Cantarella et al., 2015).

By contrast, the theoretical results for coordinated traffic signal controls are limited. Koshi (1975, 1989) showed the fundamental relationship between cycle length and delay of a signalized road under the simplification assumptions. A similar but more general result was recently reported by Jin and Yu (2015) for a stationary homogeneous ring road based on an explicit formula for a macroscopic fundamental diagram. Using a similar approach, Daganzo and Lehe (2016) explored optimal traffic coordination schemes (i.e., offsets) for different traffic density levels. However, these

* Corresponding author. Tel.: +81-3-5452-6098 (ext. 58171); Fax: +81-3-5452-6420.
*E-mail addresses:* wadaken@iis.u-tokyo.ac.jp (Kentaro Wada)., kuwahara@plan.civil.tohoku.ac.jp (Masao Kuwahara).

2352-1465 © 2017 The Authors. Elsevier B.V. All rights reserved.
Peer review under responsibility of the scientific committee of the 22nd International Symposium on Transportation and Traffic Theory.
10.1016/j.trpro.2017.05.035

theoretical studies can only examine one signal control parameter at a time, and the relationships between the different optimal control parameters are not completely understood.

To fulfill the gap between the simulation and theoretical approaches, developing mathematical programming models capable of providing an (exact) optimal solution is necessary; this is because the solution serves as a performance bound in addition to providing insight into the fundamental characteristics of coordinated signal controls. In this direction, the suitable modeling of (i) traffic flow along a signalized arterial road and (ii) various signal constraints within an optimization framework are essential.

To address the first issue, the kinematic wave (KW) model has historically been employed to describe traffic dynamics along a signalized arterial road with several intersections. Lo (1999, 2001), the more noticeable studies, formulated a coordinated signal control problem as a mixed integer linear programming (MILP) using the cell transmission model (CTM). Han et al. (2016a) also formulated a similar control problem based on the variational theory (VT) of the KW model (Daganzo, 2005a,b). Owing to the fact that the VT approach allows link-based modeling and is free from unintended vehicle holding, the number of binary variables in MILP using CTM can be substantially reduced. However, the VT-based model does not consider the stochasticity in the traffic flow. This is in sharp contrast to the isolated traffic signal optimization wherein expected delay formulae have been established (Cheng et al., 2016). Residual queues due to stochastic arrivals could largely affect delays, particularly under the near saturation condition; thus, the optimality of the resulting control parameters by a deterministic model may not be guaranteed.

The second issue regarding signal constraints is as important as traffic flow modeling because the ways of formulating the signal constraints strongly affects the complexity of the signal control optimization. Within MILP, Lo (2001) used common control parameters (e.g., green splits, offsets) as the control variables and directly modeled the relationships between the parameters. Although this approach is straightforward, the resulting formulation is quite complex because a number of *if-then rules* need to be linearized by introducing additional binary variables. In addition, the lost times were not considered despite their importance. Lin and Wang (2004) treated the lost times as penalty terms of the objective function. However, this approach failed to take into account the queueing phenomenon at traffic signals.

This study first proposes a simpler mathematical programming formulation for deterministic coordinated traffic signal control. In our model, traffic dynamics is modeled using the variational theory of KW model, and the constraints on a signal control pattern are described as a set of constraints on a certain network. The resulting MILP has a clear network structure and the requires fewer binary variables and constraints as compared with those in existing formulations. As an extension of the basic problem, we introduce stochastic boundary conditions into the VT. By exploiting the network structure of the problem, we develop an accurate and efficient approximation method of expected delays as well as a solution method for the stochastic version of the signal optimization. Using a set of proposed methods, we finally examine the optimal control parameters for deterministic and stochastic coordinated signal controls and discuss their characteristics.

The remainder of this paper is organized as follows. In Section 2, we formulate a deterministic coordinated signal control problem based on the VT. To formulate the signal constraints with minimal number of binary variables, the network representation of these constraints is presented here. In Section 3, we extend the basic problem so as to treat the stochastic fluctuations in traffic demand, and construct an accurate and efficient approximation method for the expected delays. A solution method for the stochastic version of the coordinated signal control is also developed. In Section 4, we finally examine the optimal control parameters for coordinated signal controls under both deterministic and stochastic demands using a set of methods developed in the previous sections, and discuss their characteristics. Section 5 finally concludes the study.

## 2. Coordinated signal control problem

In this section, we formulate a *deterministic* coordinated signal control problem. After discussing the problem statement in Subsection 2.1, Subsections 2.2 and 2.3 describe the objective function and constraints, respectively. We then demonstrate the overall problem and make comparisons with existing formulations in Subsection 2.4.

### 2.1. Problem setting

Consider a two-way arterial corridor with $M$ signalized intersections and $2 \times M$ crossroads, as shown in Fig.1. The set of intersections is denoted by $\mathcal{M} = \{1, 2, \dots, M\}$, and the intersections are numbered $1, 2, \dots, M$. The set of roads