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Optimal Hyperpaths With Non-Additive Link Costs

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Abstract

Non-additive link cost functions are common and important for a range of assignment problems. In particular in transit assignment, but also a range of other problems path splits further need to consider node cost uncertainties leading to the notion of hyperpaths. We discuss the problem of finding optimal hyperpaths under non-additive link cost conditions assuming a cost vector with a limited number of marginal decreasing costs depending on the number of links already traversed. We illustrate that these non-additive costs lead to violation of Bellman's optimality principle which in turn means that standard procedures to obtain optimal destination specific hyperpath trees are not feasible. To overcome the problem we introduce the concepts of a "travel history vector" and active critical, passive critical and fixed nodes. The former records the expected number of traversed links until a node, and the latter distinguishes nodes for which the cost can be determined deterministically. With this we develop a 2-stage solution approach. In the first stage we test whether the optimal hyperpath can be obtained by backward search. If this is not the case, we propose a so called "selective hyperpath generation" among hyperpaths to a (small) number of active critical nodes and combine this with standard hyperpath search. We illustrate our approach by applying it to the Sioux Falls network showing that even for link cost functions with large step changes we are able to obtain optimal hyperpaths in a reasonable computational time.

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Keywords: Non-additive costs; Bellman principle; Hyperpath; Optimal Strategies; Frequency-based Assignment

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1. Introduction

Non-additivity of cost functions along paths in a transport network is an issue for a range of applications. Energy efficiency might depend on driven distance. Regulations such as rest-times for truck drivers or delivery time windows might introduce step functions in the cost function if certain distances/travel times are exceeded. With GNSS road pricing also non-additivities might be introduced to support or discourage long distance trips. Gabriel and Bernstein (1997) provide a general discussion on the need for non-additive costs in traffic assignment and Chen and Nie (2013) extend this. As noted also in Han and Lo (2004), an application where non-linear costs are arguably most common are public transport fares. In most systems the traveler has to pay a fee to enter the system plus additional zone or distance depending fares. In case of distance-depending fares these are usually degressive in that the marginal fare per distance is decreasing. Combinations of flat, zonal and distance-depending fares plus additional features such as peak-hour surcharges are further common practice. To clarify, we are interested in this paper in cases where the fare is not predetermined by the origin and destination but depends on the actual path chosen. In Japan many fare structures comply with this criteria. Furthermore, besides fares, we might argue that congestion aspects, again particular transit congestion, also fulfill our problem description. Travelers on crowded trains often do not mind standing for one or a few stops but would make a significant effort to avoid having to stand over longer distances. In this case, and in opposite to the fare case, we would expect the cost function to be disproportionally ascending with distance though.

In case travelers can be assigned to single paths, non-additivity might be dealt with in an exact manner and network loading is fairly straightforward. However, in case the network is large and one has to consider that travelers en-route react to delays by changing their route, non-additivity forms an important challenge as we will illustrate and discuss in this paper. Considering uncertainty in path choice due to potential delays has led to the concept of hyperpaths where travelers' choices at nodes as to which link to take from an attractive set are determined by the order in which events (such as arrival of buses or trains) are unfolding. Specifically for transit applications this concept has led to a large body of literature on frequency-based assignment with consideration of passenger strategies. Similarly, the idea of hyperpath-based route choice has been applied to road network assignment (Bell, 2009; Bell et al, 2012; Ma and Fukuda, 2016) where hyperpaths might be obtained due to "random" link choices depending on, for example, which exiting link has a green traffic signal at the time a traveler arrives at a junction.

Furthermore, frequency-based transit assignment models are still the main tool to obtain line loads in large scale networks. Especially if one wants to test line load sensitivities to changes in fare distance levels or consider the aforementioned non-additive congestion effects, improved frequency-based assignment methods are required. This motivates this paper, though in the work present here we emphasize the general application of our approach to hyperpath-based traffic or transit assignment without paying attention to transit-specific issues such as walking between platforms, dwell times or capacity constraints.

The remainder of this paper is organized as follows. The next section will review related literature. In Section 3 we introduce our fairly extensive notation before Section 4 defines the problem we address more formally. We then illustrate the resulting assignment problem and classify three types of hyperpath searches depending on the fare structures in Section 5. Section 6 then lays the foundation for our solution approach that is presented in Section 7. Section 8 illustrates the approach with a case study before Section 9 concludes the paper and discusses extensions required for practical applications.

2. Literature on (hyperpath-based) assignment with non-linear link costs

A range of literature on non-additive costs has considered obtaining path costs as given and looked at equilibrium assignment approaches focusing on convergence issues given that convexity of costs might not be guaranteed (Wong et al, 2001; Han and Lo, 2004). Recently, within transit assignment, Chin et al (2016) also looked at equilibrium assignment with non-additive costs. Their focus has been further on policy implications by illustrating the impact of non-linear fares on revenue and ridership with a Toronto case study.

Our focus is the step before assignment, that is the determination of the optimal hyperpaths themselves for which there is much less literature. Finding a single shortest path in a network with one cost criteria does not constitute a significant problem as additivity of costs is not a criteria for a range of algorithms (Ahuja et al, 1993). A set of