Engineering Analysis with Boundary Elements 37 (2013) 673–682

Contents lists available at SciVerse ScienceDirect

Engineering Analysis with Boundary Elements

journal homepage: www.elsevier.com/locate/enganabound

The MLPG analyses of large deflections of magnetoelectroelastic plates

J. Sladek [a,]*, V. Sladek [a], S. Krahulec [a], E. Pan [b]

[a] *Institute of Construction and Architecture, Slovak Academy of Sciences, 84503 Bratislava, Slovakia*
[b] *Department of Civil Engineering, University of Akron, Akron, OH 44325-3905, USA*

ARTICLE INFO

Article history:
Received 20 November 2012
Accepted 5 February 2013

Keywords:
Local integral equations
von Karman plate theory
Time-harmonic load
MLS approximation
Sensor
Actuator

ABSTRACT

The von Karman plate theory of large deformations is applied to express the strains, which are then used in the constitutive equations for magnetoelectroelastic solids. The in-plane electric and magnetic fields can be ignored for plates. A quadratic variation of electric and magnetic potentials along the thickness direction of the plate is assumed. The number of unknown terms in the quadratic approximation is reduced, satisfying the Maxwell equations. Bending moments and shear forces are considered by the Reissner–Mindlin theory, and the original three-dimensional (3D) thick plate problem is reduced to a two-dimensional (2D) one. A meshless local Petrov–Galerkin (MLPG) method is applied to solve the governing equations derived based on the Reissner–Mindlin theory. Nodal points are randomly distributed over the mean surface of the considered plate. Each node is the centre of a circle surrounding it. The weak form on small subdomains with a Heaviside step function as the test function is applied to derive the local integral equations. After performing the spatial MLS approximation, a system of algebraic equations for certain nodal unknowns is obtained. Both stationary and time-harmonic loads are then analyzed numerically.

1. Introduction

Magnetoelectroelastic (MEE) materials have found much application as sensors and actuators for the purpose of monitoring and controlling the response of structures. The MEE layers are frequently embedded into laminated composite plates to control the shape of plates. The magneto-electric forces give raise to strains that could reduce the effects of the applied mechanical load. Hence, advanced structures can be designed using less material and hence less weight. Pan [1] and Pan and Heyliger [2] presented analytical solutions for the analysis of simply supported MEE laminated rectangular plates, under static deformation and free vibration. Recently, Wu et al. [3] extended the Pagano solution for the three-dimensional (3D) plate problem to the analysis of a simply supported, functionally graded rectangular plate under MEE loads. Liu and Chang [4] studied the vibration of a MEE rectangular plate. To the authors' knowledge, little work has been carried out on the geometrically nonlinear problems occurred at large plate deformations. So far, only one paper [5] is dealing with the nonlinear behaviour of a MEE plate, where a simplified analytical solution was given for a thin simply supported MEE plate under a large deformation. The Kirchhoff plate bending theory with vanishing shear stresses was utilized.

The conventional von Karman-type nonlinear field equations for the finite deflection of plates are based on the Kirchhoff-Love assumption and follow inevitable coupling between in-plane and bending deformations, which makes analytical solutions difficult. Therefore, a simplified governing field equation known as the decoupled Berger equation [6] is also used for geometrically nonlinear deformation of plates. The Berger equation could be a fairly good approximation to the corresponding rigorous solution, provided that the in-plane displacements are constrained at the boundary [7]. Among the early proposals for analysing the final deflection of thin plates is the work by Kamiya and Sawaki [8]. The first finite element analysis of geometrically nonlinear plate behaviour using a Mindlin formulation was given by Pica et al. [9]. The boundary element method (BEM) was applied by Lei et al. [10] in the geometrically nonlinear analysis of laterally loaded isotropic plates, taking into account the effect of transverse shear deformation. A nonlinear analysis of Reissner plates by BEM was given by Qin [11]. Wen et al. [12] analyzed the post-buckling of Reissner plates. Recently, a strong formulation with multiquadric radial basis function was applied to the isotropic Reissner–Mindlin plates with geometrical nonlinearity [13].

The solution of the boundary or initial boundary value problems for MEE plates with large deformations requires advanced numerical methods due to the high mathematical complexity. Besides the well established finite element method (FEM) and the BEM [14,15], the meshless methods provide an efficient and popular alternative to these traditional computational methods.

* Corresponding author. Tel./fax: +421 2 54773548.
E-mail address: jan.sladek@savba.sk (J. Sladek).

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http://dx.doi.org/10.1016/j.enganabound.2013.02.001

Focusing only on nodes or points instead of elements used in the conventional FEM, meshless approaches have certain advantages. The elimination of shear locking in thin walled structures by FEM is difficult and the developed techniques are less accurate. The moving least-square (MLS) approximation ensures C^1 continuity which satisfies the Kirchhoff hypotheses. The continuity of the MLS approximation is given by the minimum between the continuity of the basis functions and that of the weight function. So continuity can be tuned to a desired degree. Previous results showed excellent convergence for linear problems [16–18], however, up to now the formulation has not been applied to large deflection of MEE plate problems. Recently, new class of hybrid/mixed finite elements, denoted as HMFEM-C, was developed for modelling two-dimensional (2D) problems in MEE materials [19]. These elements were based on assuming first the independent strain, electric and magnetic fields, and then collocating them with the strain, electric and magnetic fields derived from the primal variables (mechanical displacement, electric and magnetic potentials) at certain selected points inside each element. The newly developed elements showed significantly higher accuracy than the primal elements for the electric, magnetic as well as the mechanical variables, comparable to the accuracy from the meshless approach [19]. Up to date, however, these hybrid finite elements have not been applied to plate bending problems.

One of the most rapidly developed meshfree methods is the meshless local Petrov–Galerkin (MLPG) method [20]. The MLPG method has attracted much attention in the past decade and it has been successfully applied also to plate problems [21–24]. The modelling of piezoelectric plates has been done by the MLPG too [25,26].

This paper proposes a nonlinear (or large-deformation) model for the MEE thick plate under a static and time-harmonic mechanical load and a stationary electromagnetic load. It is the first effort to develop the meshless method based on the local Petrov–Galerkin weak-form to solve dynamic problems for thick MEE plates under a large deformation described by the Reissner–Mindlin theory. The electric and magnetic field components are assumed to be zero in the in-plane directions of the plate. A quadratic power-expansion of the electric and magnetic potentials in the thickness direction of the plate is considered. The bending moment, normal and shear force expressions are obtained by integration through the plate for the considered constitutive equations. The Reissner–Mindlin governing equations of motion are subsequently solved for a time-harmonic plate bending problem. The Reissner–Mindlin theory reduces the original 3D thick plate problem to a 2D problem. In our meshless method, nodal points are randomly distributed over the neutral plane of the considered plate. Each node is the centre of a circle surrounding this node. The weak form on the small subdomains with a Heaviside step function as the test function is applied to derive local integral equations. Applying the Gauss divergence theorem to the weak form, the local boundary-domain integral equations are derived. The nonlinear terms occurred in the normal and shear forces are considered iteratively in the full-load algorithm. After performing the spatial MLS approximation, a system of algebraic equations for certain nodal unknowns is obtained. Numerical examples are presented and discussed to show the accuracy and the efficiency of the present method.

2. Local integral equations for magnetoelectroelastic plates

We consider a plate of total thickness h with homogeneous MEE material properties with its mean surface occupying the domain Ω in the plane (x_1,x_2). The axis $x_3 \equiv z$ is perpendicular to the mid-plane (Fig. 1) with the origin at the bottom of the plate.

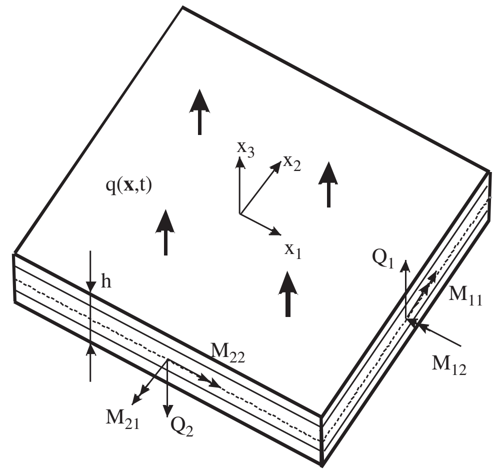

Fig. 1. Sign convention of bending moments and forces for a plate.

The Cartesian coordinate system is introduced such that the bottom and top surfaces of plate is placed in the plane $z=0$ and $z=$h, respectively. Using the von Karman theory of large deflection of plates described by the Reissner–Mindlin theory, the Lagrangian strain displacement relations are given by Pica et al. [9] and Azizian and Dawe [27]:

$$\begin{aligned}
\varepsilon_{11}(\mathbf{x},x_3,\tau) &= u_{0,1} + (z-z_0)w_{1,1}(\mathbf{x},\tau) + \frac{1}{2}(w_{3,1}(\mathbf{x},\tau))^2, \\
\varepsilon_{22}(\mathbf{x},x_3,\tau) &= v_{0,2} + (z-z_0)w_{2,2}(\mathbf{x},\tau) + \frac{1}{2}(w_{3,2}(\mathbf{x},\tau))^2, \\
\varepsilon_{12}(\mathbf{x},x_3,\tau) &= \frac{1}{2}(u_{0,2}+v_{0,1}) + \frac{1}{2}(z-z_0)\left[w_{1,2}(\mathbf{x},\tau)+w_{2,1}(\mathbf{x},\tau)\right] \\
&\quad + \frac{1}{2}w_{3,1}(\mathbf{x},\tau)w_{3,2}(\mathbf{x},\tau), \\
\varepsilon_{13}(\mathbf{x},\tau) &= [w_1(\mathbf{x},\tau)+w_{3,1}(\mathbf{x},\tau)]/2, \\
\varepsilon_{23}(\mathbf{x},\tau) &= [w_2(\mathbf{x},\tau)+w_{3,2}(\mathbf{x},\tau)]/2. \qquad (1)
\end{aligned}$$

where z_0 indicates the position of the neutral plane. For a homogeneous plate it is located in the geometrical mid-plane. In-plane displacements in x_1- and x_2-directions are denoted by u_0 and v_0. Rotations around x_2- and x_1-axes are denoted by w_1 and w_2, and w_3 is the out-of-plane deflection.

The constitutive equations for the stress tensor, electrical displacement and magnetic induction of the MEE materials are given by Nan [28]:

$$\sigma_{ij}(\mathbf{x},x_3,\tau) = c_{ijkl}\varepsilon_{kl}(\mathbf{x},x_3,\tau) - e_{kij}E_k(\mathbf{x},x_3,\tau) - d_{kij}H_k(\mathbf{x},x_3,\tau), \qquad (2)$$

$$D_j(\mathbf{x},x_3,\tau) = e_{jkl}\varepsilon_{kl}(\mathbf{x},x_3,\tau) + h_{jk}E_k(\mathbf{x},x_3,\tau) + \alpha_{jk}H_k(\mathbf{x},x_3,\tau), \qquad (3)$$

$$B_j(\mathbf{x},x_3,\tau) = d_{jkl}\varepsilon_{kl}(\mathbf{x},x_3,\tau) + \alpha_{kj}E_k(\mathbf{x},x_3,\tau) + \gamma_{jk}H_k(\mathbf{x},x_3,\tau), \qquad (4)$$

where $\{\varepsilon_{ij},E_i,H_i\}$ is the set of the secondary field quantities (strain, intensity of electric field, intensity of magnetic field) which are expressed in terms of the gradients of the primary fields, i.e., the elastic displacement vector, electric potential, and magnetic potential $\{u_i,\phi,\psi\}$. Finally, the elastic stress tensor, electric displacement, and magnetic induction vectors $\{\sigma_{ij},D_i,B_i\}$ form the set of the fields conjugated to the secondary fields $\{\varepsilon_{ij},E_i,H_i\}$. The constitutive equations correlate these two sets of fields in continuum media including the multi-field interactions.

The plate thickness is assumed to be small as compared to its in-plane dimensions. The normal stress σ_{33} is then vanishing in comparison with other normal stresses. Assuming also that the MEE materials process certain material symmetry, one can formulate the plane-deformation problems [29]. For instance, for the poling direction along the positive x_3-axis the constitutive