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Efficient evaluation of weakly/strongly singular domain integrals in the BEM using a singular nodal integration method

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ABSTRACT

In many analyses of engineering problems based on boundary element methods, a large number of regular and/or singular domain integrals must be accurately evaluated over a single domain. Evaluation of such domain integrals is very time-consuming and is frequently the main source of errors and loss of accuracy in the solutions. Previous efforts have been constantly made in order to facilitate or overcome such shortcomings. In this article, we propose novel and efficient approaches in the framework of Cartesian transformation method (CTM) and the radial integration method (RIM) that can be used for fast evaluation of numerous weakly/strongly singular two-dimensional domain integrals over a single domain. The domain integrals essentially are expressed in terms of some coefficient matrices and vectors, most of which are independent of the integrand of the domain integrals and are dependent only on the geometry. Several examples for the evaluation of weakly/strongly singular domain integrals and two examples for the flow field analysis in micro-channels are presented and the accuracy and convergence of the proposed approaches are investigated.

1. Introduction

The BEM is now extensively used for the solution of engineering and scientific problems. In the boundary element analysis of some problems such as transient or nonlinear problems, a large number of regular and/or singular domain integrals must be computed with enough accuracy and efficiency. For example, in the boundary element analysis of the transient heat conduction problem including a nonlinear domain heat source with a total number of 1000 boundary and internal nodes, 100 time steps and three iterations in each time step, $1000\times 100\times 3\times 2=600{,}000$ domain integrals must be computed [1]. Only a narrow range of domain integrals can be exactly transformed into the boundary with no approximations. The domain integrals can be computed by domain discretization using internal cells. By this way, the main benefit of the BEM as a boundary method will fade in its numerical solution procedures. The domain integrals may also be computed without discretization of the domain, nonetheless; this requires representing the domain by some internal points and using global shape functions for approximating the integrand. The accurate evaluation of domain integrals is a very important issue in the BEM and is still an important area of research [2–5].

The dual reciprocity method (DRM) [6] is the most popular technique for the evaluation of the BEM domain integrals. In the DRM, some internal points should be considered. The location of internal points can be selected arbitrarily, but the shape or basis functions used for interpolation in the domain are not arbitrary. These functions should have particular solutions and they cannot be selected arbitrarily.

Ochiai presented the triple-reciprocity method [7,8] for meshless evaluation of domain integrals in the BEM. In this method, effects of domain loadings and initial conditions are interpolated using auxiliary boundary integral equations. Accurate solutions can be obtained using this method [9].

Gao presented the radial integration method [10,11] for evaluation of 2D and 3D domain integrals in the BEM. In the RIM, the domain integral is transformed into a boundary and a radial integral. The weak singularity of domain integrals is automatically treated by the RIM. Recently, Gao and Peng [3] have presented a new version of the RIM for the evaluation of higher order singular domain integrals.

Hematiyan presented the Cartesian transformation method [12,13] for evaluation of 2D and 3D domain integrals in the BEM. Khosravifard and Hematiyan extended the CTM for evaluation of domain integrals in meshless methods [14]. In the CTM, a domain integral is transformed into a boundary and a simple 1D integral. The CTM is very efficient for meshless evaluation of

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domain integrals; however, this method cannot be directly used for evaluation of singular domain integrals.

The main goal of the present work is to propose a novel method, which can be used, for fast evaluation of numerous singular domain integrals with different integrands over a single domain. The CTM and the RIM are both used in the proposed method. In the approach to be presented herein, the essential feature and a key factor is that the domain integrals are not transformed to the boundary while they are efficiently evaluated using a meshless procedure without employing any internal cells or background meshes. The interpolation shape functions used in the method can be selected arbitrarily and can have very complicated forms. By presenting some numerical examples, it is shown that the proposed technique is efficient for evaluation of weakly and strongly singular domain integrals.

The structure of the paper is organized as follows. In Section 2, a nodal integration method for evaluation of regular domain integrals is presented. In Section 3, the singular nodal integration method is developed for the evaluation of the weakly singular domain integrals, whereas the strongly singular domain integrals are given in Section 4. Numerical examples are presented and investigated in Section 5. Finally, the essential conclusions drawn from the present study are given in the last section.

2. A CTM-based nodal integration method for meshless evaluation of regular domain integrals

Suppose that we want to compute the following regular integral over the two dimensional domain Ω with $\mathbf{x}=(x,y)$

$$I=\int_{\Omega} f(\mathbf{x})d\Omega \tag{1}$$

In this section, a nodal integration method for the evaluation of regular domain integrals is presented. The method is essentially developed based on the CTM [12–14]. Firstly; the weighted values of the integrand at some internal points are found. Then, by a simple summation of those weighted values, the total value of the integral is obtained. It is assumed that the values of the function $f(\mathbf{x})$ at M_B boundary points and M_I internal points in the domain Ω are known (Fig. 1). These nodal points are denoted by $\mathbf{P}_1, \mathbf{P}_2, \cdots, \mathbf{P}_M$, where $M=M_B+M_I$. Meshless interpolation methods [15] such as the radial point interpolation method (RPIM) [16] or the moving least squares approximation method (MLS) [17] can be employed for obtaining an approximate value of the function $f(\mathbf{x})$ at an arbitrary point.

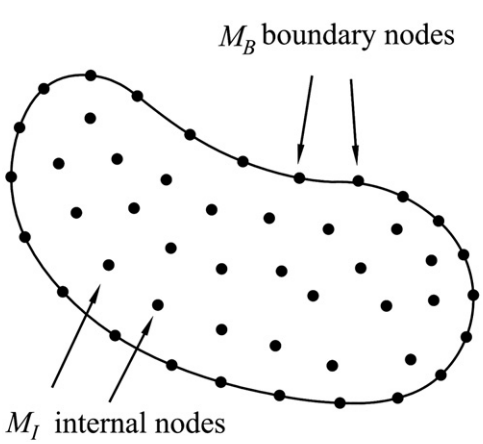

Fig. 1. Boundary and internal nodes.

By utilizing a meshless interpolation method, the function $f(\mathbf{x})$ can be expressed in the following general form [15],

$$f(\mathbf{x})=\sum_{i=1}^{M}\phi_i(x,y)f_i=\mathbf{\Phi}^{\mathrm{T}}\mathbf{f} \tag{2}$$

where the vector $\mathbf{f}$ contains the values of function $f(\mathbf{x})$ at the M nodal points while $\mathbf{\Phi}$ collects the values of the shape functions. The specific form of $\mathbf{\Phi}$ depends on the interpolation technique used. The formulation of the present work allows the use of any arbitrary interpolation method. Due to its accuracy and ease of use, the RPIM is employed in the present work. In the RPIM, radial and polynomial basis functions are used to obtain a continuous function that passes over a scattered set of data. The general form of this interpolating function is written as:

$$f(\mathbf{x})=\sum_{i=1}^{n}R_i(\mathbf{x})a_i+\sum_{j=1}^{m}p_j(\mathbf{x})b_j=\left[\mathbf{R}^{\mathbf{T}}(\mathbf{x}) \quad \mathbf{p}^{\mathbf{T}}(\mathbf{x})\right]\begin{Bmatrix}\mathbf{a}\\ \mathbf{b}\end{Bmatrix} \tag{3}$$

where R_i is a radial basis function (RBF), and p_j is a monomial in space coordinates. n is the number of nodes in the support domain of point $\mathbf{x}$, m is the number of monomials used, and a_i and b_j are unknown constants to be determined. It is worth mentioning that the RPIM has a local nature, i.e., not all of the M nodal points are used for the interpolation of the function at a point $\mathbf{x}$. Only the nodes which are close to the point $\mathbf{x}$, and are thus located in the support domain of this point, are used in the interpolation process. There are several types of RBFs which can be used in the formulation of the RPIM. In the present paper, the thin plate spline (TPS) function [15] is used:

$$R_i(\mathbf{x})=r_i^{\eta} \tag{4}$$

where r_i is the Euclidean distance between the point $\mathbf{x}$ and the ith node in the support domain, i.e.:

$$r_i=\sqrt{(x-x_i)^2+(y-y_i)^2} \tag{5}$$

η in Eq. (4) is a constant parameter, with usual values of 3.001, 4.001, or 5.001 [15]. In this work the value of η is 4.001.

The RPIM can also be used to obtain a smooth and continuous function that approximates the spatial derivative of a scattered set of data. This is done by direct differentiation of Eq. (3), i.e.:

$$\frac{\partial f(\mathbf{x})}{\partial s}=\sum_{i=1}^{n}\frac{\partial R_i(\mathbf{x})}{\partial s}a_i+\sum_{j=1}^{m}\frac{\partial p_j(\mathbf{x})}{\partial s}b_j=\mathbf{\Phi}_{,s}^{\mathrm{T}}\mathbf{f}_u, \quad s=x \text{ or } y \tag{6}$$

In Eq. (6) $\mathbf{\Phi}$ is the vector of RPIM shape functions, details of which can be found in [15]. Now the nodal integration method for meshless evaluation of the domain integral in Eq. (1) is described. Fig. 2 depicts the domain Ω inscribed in a rectangle. The domain integral can be recast as follows:

$$I=\int_c^d\left(\int_a^b f_A(\mathbf{x})dx\right)dy \tag{7}$$

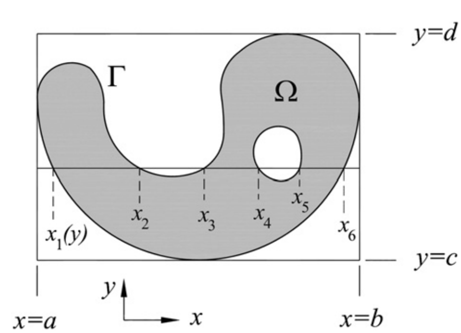

Fig. 2. A rectangle circumscribed over the domain.