Engineering Analysis with Boundary Elements 37 (2013) 699–707

Contents lists available at SciVerse ScienceDirect

Engineering Analysis with Boundary Elements

journal homepage: www.elsevier.com/locate/enganabound

A coupled BEM-stiffness matrix approach for analysis of shear deformable plates on elastic half space

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ARTICLE INFO

Article history:
Received 29 March 2012
Accepted 21 December 2012

Keywords:
Boundary element method
Elastic half space
Plates
Stiffness matrix
Reissner plates

ABSTRACT

In this paper, a new direct Boundary Element Method (BEM) is presented to solve plates on elastic half space (EHS). The considered BEM is based on the formulation of Vander Weeën for the shear deformable plate bending theory of Reissner. The considered EHS is the infinite EHS of Boussinesq–Mindlin or the finite EHS (with rigid end layer) of Steinbrenner. The multi-layered EHS is also considered. In the present formulation, the soil stiffness matrix is computed. Hence, this stiffness matrix is directly incorporated inside the developed BEM. Several numerical examples are considered and results are compared against previously published analytical and numerical methods to validate the present formulation.

1. Introduction

In structural designs, soil-structural interaction problem is always point of research where good representation is necessary. One of the most common methods in design companies is the Winkler spring method [1]. It is based on representing the soil as individual springs. Such a model assumes the displacement of soil medium at any surface point to be directly proportional to the applied stress and independent of stresses applied to other locations. The displacement occurs immediately under the loaded area; whereas displacement outside this region is zero. The Winkler method is mainly dependent on accurate determination of the coefficient of sub-grade reaction [2]. There is a large range of sub-grade reaction values produced from many methods such as experienced charts and methods based on the theory of elasticity [2]. The Winkler method does not consider coupling between springs through soil layers but relies on the attached footing stiffness [2]. Moreover, the Winkler method does not take soil layering and most engineers rely on the surface layer properties or properties of an equivalent layer. Although this method dates many decades, it is still used until now because of its simplicity.

The two-parameter elastic models have been developed as refined soil representations. These models use pre-defined two independent elastic constants. Some of these models provide mechanical interaction between Winkler individual springs using elastic membrane, elastic beams, or elastic layers that carry soil shear deformations. Examples of such models are the work of Filonenko-Borodich [3,4], Hetenyi [5], Pasternak [6] and Kerr [7]. Other two-parameter models are based on simplified assumptions to the original elastic continuum model, such as the models of Reissner [8] and Vlasov and Leontiev [9].

It has to be noted that, the sub-grade reaction modulus and other soil parameters are not mechanical soil properties [2] but they depend on the shape and the load pattern of the loaded area.

The ACI committee [10] suggested using an elastic half space technique with the Boussinesq theory instead of the Winkler model for accurate modeling. Unlike the Winkler and the two-parameter models, the elastic half space method uses data obtained from geotechnical investigations.

There are many models that treat soil as an elastic half space. Among them are the models of Boussinesq [11] and Mindlin [12], which consider the soil as an elastic, isotropic, homogenous, and infinite half space. The Steinbrenner elastic half space model [13], on the other hand, considers presence of a rigid layer under the considered surface soil finite layer. Another method that analyzes the soil under plates is the finite layer method [14] in which the soil is divided into several horizontal layers.

Plates resting on elastic half space are studied with the finite element method and the boundary element method besides presence of some analytical solutions. This is considering thin and thick plate theories. The application of the BEM to plate bending problems modeled using the thin plate theory was introduced by Bézine [15] and Stern [16]. Vander Weeën [17] derived a BEM for plate bending problems based on the shear deformable plate theory according to Reissner [18]. In the last 20

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http://dx.doi.org/10.1016/j.enganabound.2012.12.005

years, the formulation of Vander Weeën [17] became a standard formulation among researchers due to its stability and versatility. The edited book of Aliabadi [19] contained several advanced developments in BEM for plate bending problems using Vander Weeën formulation. Rashed [20] extended the formulation of Vander Weeën [17] to solve practical rafts on Winkler foundation.

Several researches have discussed the analysis of plates on elastic half space. Selvadurai [11], Bowels [13], and Das [21] proposed different methods of elastic analysis of soil–plate interaction. Hemsley [22] proposed an elastic solution for axisymmetrically loaded circular plate with free and clamped edges on Winkler springs and on a half-space. Chen and Peng [23] demonstrated a finite element computation of plates on elastic half space for various base-models. Timoshenko and Goddier [24] also discussed few methods for soil–plate analysis based on the theory of elasticity. Wang et al. [25] used bending of plates on an elastic half-space analyzed by isoprametric finite elements. Wardle and Fraser [26] carried out a finite element analysis of a plate on a layered cross-anisotropic elastic half space. They also discussed a numerical analysis of rectangular plates on layered elastic half space in Ref. [27]. Ta and Small [14] carried out an analysis of plates modeled by FEM on finite layered half space with full analysis and another analysis uses some approximations. Stavridis [28] proposed a simplified analysis of layered soil-structure interaction. Wang et al. [12] provided an analysis of rectangular thick plates on an elastic half space using Ritz method. Wang et al. [29] demonstrated a plate on layered elastic half space analyzed by a semi-analytical and semi-numerical method.

Considering the boundary element modeling for plates on elastic half space, Syngellakis and Bai [30] discussed the application of the boundary element method to thin plate on the Boussinesq half space. Xiao [31] analyzed thick plates on elastic half space using special form of the indirect boundary integral formulation where in his analysis; results are obtained in terms of two Hu functions [32] with no reference to physical variables. The formulation of Xiao [31] produces hyper-singular kernels in the integral equations. Therefore, in Ref. [31], the collocation points are placed outside the boundary together with using constant elements to avoid hyper-singular integrals. This leads to limit the application of the method presented in Ref. [31] to small problems of no practical use.

This paper presents a new practical technique of using the Boundary Element Method (BEM) to solve plates on elastic half space. The considered BEM is based on the formulation of Vander Weeën [17] for the shear deformable plate bending theory. The considered EHS is the infinite EHS of Boussinesq–Mindlin or the finite EHS (with rigid end layer) of Steinbrenner. The multi-layered EHS is also considered. In the present formulation, the soil stiffness matrix is computed. Hence, this stiffness matrix is directly incorporated inside the developed BEM.

Several numerical examples are presented and results are compared against previously published analytical and numerical methods to validate the present formulation.

2. Stiffness matrix for multi-layered elastic half space

In this section, the stiffness matrix of the elastic half space is formed and modified to be ready to be fit into the proposed BEM formulation in the next section.

2.1. Elasticity solutions

In this section, elastic solutions of Boussinesq, Mindlin and Steinbrenner are reviewed. The EHS boundary (surface under the plate) is divided into Nc area segments at which the displacement is required to be computed in any segment due to loading at the origin. The following sub-sections compute the flexibility matrix of the overall EHS divisions.

2.1.1. Bousinessq solution

The displacement $w_{(X,Y)}$ of a point lying on the surface of an elastic, isotropic, homogenous and infinite thickness half space due to a concentrated load P acting at the origin (0, 0) is [11]

$$w_{(X,Y)} = \frac{(1-\nu)P}{2\pi G r} \tag{1}$$

in which $G = E/2(1+\nu)$ is the shear modulus, E is the modulus of elasticity, ν is the Poisson's ratio and

$$r = \sqrt{X^2+Y^2}.$$

In order to avoid singularity under loading when computing $w_{(0,0)}$, the concentrated loading is replaced by equivalent pressure of intensity q over circular area of radius a [11], to give

$$w_{(0,0)} = \frac{(1-\nu)qa}{G} \tag{2}$$

2.1.2. Mindlin solution

Similar to Bousinessq solution, Mindlin solution considers similar equation to Eq. (1) for the surface displacement $w_{(X,Y)}$ at distance $r=\sqrt{X^2+Y^2}$. However, displacement under load is computed by integrating the equivalent uniform load over rectangular area $(B \times L)$ to give [12]

$$w_{(0,0)} = \frac{P}{8\pi G(1-\nu)B}\left((3-4\nu)\left(\beta \ln\left(\frac{1+\sqrt{1+\beta^2}}{\beta}\right)+\ln\left(\beta+\sqrt{1+\beta^2}\right)\right) + (5-12\nu-8\nu^2)\left(\beta \ln\left(\frac{1+\sqrt{1+\beta^2}}{\beta}\right)+\ln\left(\beta+\sqrt{1+\beta^2}\right)\right)\right) \tag{3}$$

in which $\beta = B/L$.

It has to be noted that, displacement values computed from both Boussinesq and Mindlin solutions could be multiplied by a factor to account for the presence of rigid layer at limited depths [21].

2.1.3. Steinbrenner solution

It considers the displacement w of a rectangular loaded area of dimension $B \times L$ on the surface of elastic half space having a finite soil layer underneath the plate. Such a soil layer is above a rigid layer. The displacement is computed based on theory of elasticity as follow [13]:

$$w = qB\frac{1-\nu^2}{E} m\left(I_1 + \frac{1-2\nu}{1-\nu} I_2\right) I_F \tag{4}$$

where q is the equivalent uniform applied stress over the loaded area $(B \times L)$, E is the modulus of elasticity and ν is the Poisson's ratio. I_1 and I_2 are influence factors computed using equations given by Steinbrenner [13] as follow:

$$I_1 = \frac{1}{\pi}\left(M \ln \frac{\left(1+\sqrt{M^2+1}\right)\sqrt{M^2+N^2}}{M\left(1+\sqrt{M^2+N^2+1}\right)} + \ln \frac{\left(M+\sqrt{M^2+1}\right)\sqrt{1+N^2}}{M+\sqrt{M^2+N^2+1}}\right) \tag{5}$$

$$I_2 = \frac{N}{2\pi}\tan^{-1}\left(\frac{M}{N\sqrt{M^2+N^2+1}}\right) \tag{6}$$

in which $M = L/B$, $N = H/B$ and H is the height of the soil layer above the rigid layer. I_F is the influence factor depending on plate embedment depth D, in this work, I_F is taken to be $=1$ as all considered plates in this paper are located on the EHS surface. m is the number of corners contributing to displacement w. At the