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Current distribution in circular planar coil

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ABSTRACT

The current distribution over the cross section of a planar circular coil is calculated by a Fredholm integral equation technique. An external applied current source is driving the current. The integral equation technique is applied over a two-dimensional cross section of the coil while considering infinitesimally thin windings. The coil windings are divided into equally sized one-dimensional elements. The resulting algebraic system is solved numerically. For low frequencies, the current distribution follows the $1/r$ behavior. As the frequency increases, the influence of the proximity effect is taken into account. Different cases are studied examining the intensity of these effects on the current distribution as the number of turns, the width of the windings, and the spacing between the turns are varying.

1. Introduction

Spiral planar inductors are widely used in amplifiers, oscillators, switches etc with varying shape and size. These inductors are made from thin metallic layers deposited on printed circuit boards, ceramic substrate or on silicon integrated circuits. In Fig. 1, we can see different types of spiral coils such as square octagonal and circular coils. Various techniques are used to calculate their inductance, resistance and quality factor [1]. On the other hand, little is mentioned about the current density distribution in planar spiral inductors.

At the same time, different approaches can be found in the literature for estimating the current density distribution in parallel conductors. These techniques consider integral [2,3] or apply a more empirical/engineering approach using partial inductance–resistance matrices [4].

In addition, integral equations are reported to have been used in order to solve vector magnetic potential and current distribution problems in linear or axi-symmetrical strips. Integral representations have been used in [5,6] to solve the current distributions problems on various axi-symmetrical problems.

Similar problems have also been attacked by Kroot et al. with a particular application to magnetic resonance imaging gradient coils [3,7–9]. In the above mentioned researches, Legendre polynomials are used as a basis function for the solutions of the integral equations. In [7–9] the Galerkin method is applied. Moreover, the importance of proximity and edge effects, as well as the low-frequency non-uniformity of the current distribution in the case of axy-symmetrical coils have been underlined.

In the present paper, the proposed method is used to calculate the current density in the cross-sectional area of a planar circular coil. The thickness of the coil is considered infinitesimally small compared to the radii of the windings and to the skin depth for the frequencies under consideration. As the frequency increases, the proposed method loses apparently its validity as the skin depth becomes comparable to the thickness of the coil. For the range of frequencies used in this study though, the assumption that the skin depth is bigger than the thickness of the coil, is considered valid.

This semi-analytical method followed in this very research introduces a Fredholm integral equation technique applied previously in [10,11]. The solutions provided are in the form of multiple integrals of modified Bessel functions. The system of equations has been applied on a one-dimensional cross section of the coil as the thickness is neglected. The Gauss elimination method has been used to solve the algebraic system. The results confirm the non-uniform $1/r$ behavior of the current distribution for low frequencies. For higher frequencies, the proximity effect becomes prominent and the current distribution is heavily altered.

The main purpose of the ongoing investigation is to find out the temperature distribution of spiral inductors with micrometer dimensions in an integrated circuit. It was observed experimentally using infrared thermography that the temperature increase is not negligible even when the total power was in the mW range [12]. This numerical model had been developed in order to be used in a 2-dimensional heat transfer model of a circular–spiral-shaped heat

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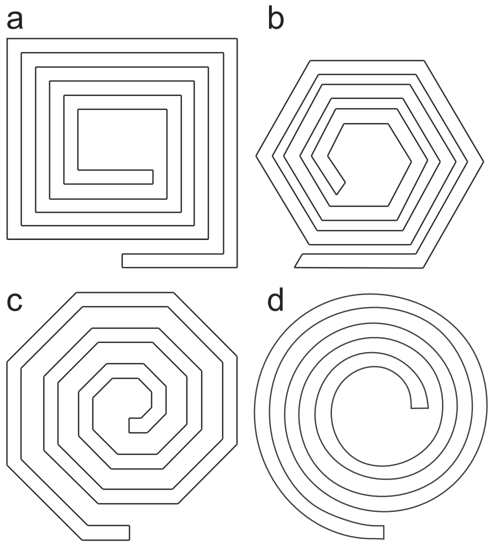

Fig. 1. Planar inductors - (a) square, (b) exagonal, (c) octagonal, and (d) spiral.

source. From the calculated current density it is quite straightforward to obtain the spatial distribution of the joule losses.

2. Integral equation

A circular planar inductor is assumed. It consists of N turns which are concentric. In Fig. 2, we can see a layout of a 2-turn circular planar coil. The inductor is triggered by an imposed source current $I^{(n)}$, which is the complex value of the source current on turn $^{(n)}$, with $n=1,\ldots,N$. The total prescribed current of each turn is equal to I_0. The introduced signal generates a magnetic field which in turn introduces eddy currents. These eddy currents alter the current distribution and therefore the signal introduced and the magnetic field. The turns of the inductor are considered to be very thin compared to other dimensions of the coil, and thus the current density and the related electric field are in the xy-plane. The magnetic field caused by the imposed current, and also by the eddy currents, is directed along the z-axis for points in the xy-plane.

The current flows in circles sharing the same center and thus the problem is circular symmetric and, the current density can be described as

$$\vec{J} = J(r)\overrightarrow{u_\theta}, \tag{1}$$

where r, θ are the polar coordinates (Fig. 2). Remark that all quantities are in phasor notation unless otherwise mentioned.

The equation describing the function of eddy currents is

$$\nabla \times \vec{E} = -j\omega\vec{B} = -j\omega(\nabla \times \vec{A}), \tag{2}$$

where ω represents the angular frequency, $\vec{A}$ is the vector potential, $\vec{E}$ is the electric field, and $\vec{B}$ is the magnetic induction. From the previous equation one can easily get by integration:

$$\vec{E} = -j\omega\vec{A} + \nabla\phi, \tag{3}$$

where $\nabla\phi$ is an integration constant, and ϕ is the scalar potential function related to the imposed electrical field $\overrightarrow{E_0}$, caused by the imposed currents $I^{(\mathrm{n})}$:

$$\overrightarrow{E_0} = \nabla\phi. \tag{4}$$

Due to circular symmetry the potential related function ϕ can only depend on r. It is also known that the imposed current

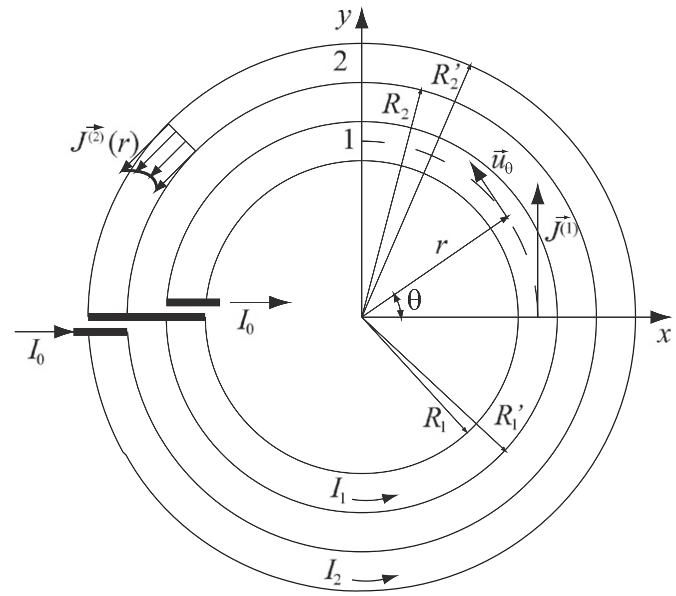

Fig. 2. Circular planar coil layout.

density is related to the electric field caused by that signal as

$$\overrightarrow{J_0^{(n)}}(r) = \sigma\overrightarrow{E_0^{(n)}}(r), \tag{5}$$

where n is the number of the turn under investigation, $\overrightarrow{J_0^{(n)}}(r)$ is the imposed current density along the cross section of the turn n, $\overrightarrow{E_0^{(n)}}(r)$ is the induced electric field at the same point $\vec{r}$ on the turn $^{(n)}$, and σ is the electric conductivity of the coil.

In addition, the voltage drop along one turn, at a distance r from the center of the coil in the DC case, is described by the following equation:

$$V^{(n)}(r) = \oint_r \overrightarrow{E_0^{(n)}}(r)\overrightarrow{dl} = E_0^{(n)} 2\pi r = C^{(n)}. \tag{6}$$

The voltage drop over each turn $^{(n)}$ remains constant and is equal to $C^{(n)}$ as the radius r varies between R_n and R'_n. R_n and R'_n represent each turn's inner and outer radius, respectively. Furthermore, the length of each turn's arc is considered equal to the perimeter of the circle on which it is drawn.

Although $V^{(n)}(r)$ is a complex representation of a sinusoidal varying quantity (time dependent) for frequencies different than zero, Eq. (6) remains valid as in the DC case, because ω does not appear in (6).

Combining the previous Eqs. (5) and (6), one can get the following relation:

$$\overrightarrow{J_0^{(n)}}(r) = \frac{\sigma C^{(n)}}{2\pi r}\overrightarrow{u_\theta}. \tag{7}$$

Besides, the spatial distribution of the imposed current for frequencies different than zero ($f \neq 0$) is the same as for the zero-frequency case.

The imposed currents $I^{(n)}$ are equal to

$$I^{(n)} = d\int_{R_n}^{R'_n} \overrightarrow{J_0^{(n)}}\overrightarrow{dr} = d\sigma C^{(n)}\int_{R_n}^{R'_n} \frac{1}{2\pi r}\, dr \Rightarrow$$

$$C^{(n)} = \frac{2\pi I^{(n)}}{d\sigma \ln \frac{R'_n}{R_n}}, \tag{8}$$

where n is the number of the turn into consideration, R_n is the inner radius of the turn and R'_n is the outer radius of the turn and d the thickness of the coil assumed to be much smaller than the radius. It follows that the imposed electrical field is given by the