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Quantitative analysis of interaction range in vehicular flows

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Abstract

Applying mathematical theory of functional convolutions we quantitatively detect a range of interaction forces inside ensembles of moving vehicles. With help of a certain specifically-calibrated function describing deflections between empirical multi-clearance distributions and distributions derived for Poissonian systems of uncorrelated agents we estimate the number of neighboring vehicles whose movements are significantly influenced (in a statistical sense). Furthermore, we demonstrate how the estimated interaction-range varies with traffic flux or traffic density. The obtained results convincingly confirms that vehicular dynamics is definitely not of short-ranged nature but, in contrast, that mutual inter-vehicular interactions exist among more succeeding cars.

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Keywords: vehicular traffic; multi-headway distribution; interaction range

1. Introduction

Generally, traffic systems represent granular ensembles whose intelligent agents interact with a certain set of their neighbors. Although such an interaction is not directly measurable (then, a fortiori, neither is interaction range) some recent works (see Helbing (2004, 2006); Krbalek (2008, 2009); Jin (2009); Treiber (2009); Chen (2010); Krbalek (2013, 2015)) have revealed a way how to approach to a realistic quantitative description of vehicular micro-dynamics. Indeed, in the articles Krbalek (2008, 2009, 2013, 2015) authors have demonstrated that one-dimensional thermal gas, whose particles interact via a repulsion potential depending on reciprocal value of distance among succeeding cars, represents a suitable theoretical model that is capable to reproduce vehicular flows on a microscopic scale surprisingly precisely. However, most microscopic traffic-models (including the above-referred thermal model) are based on the hypothesis (see Treiber (2013); Helbing (2001)) that a direct interaction exists between neighboring vehicles only. Is, as supposed in most recent traffic models, such an interaction really short-ranged? Or, in contrast, a chosen agent interact with more his neighboring agents. The main objective of this paper is to decide (by means mathematical theories) how many closest cars influence decision-making procedures of a driver. To be specific, we will introduce a mathematical methodology for deciding how many succeeding cars influence a driver in a given traffic situation. The method presented is based on an analysis of so-called *multi-clearances* in empirical traffic data provided by the Road and Motorway Directorate of the Czech Republic.

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A note: To prevent any misinterpretation, we remark that the respective descriptions for ranges of interactions (short/middle/long ranges) reflect how many neighboring elements (agents, particles, vehicles) interact with a chosen element. If a movement of the chosen agent is influenced by an immediately neighboring agent only, we will call such an interaction a short-ranged one. If there exist interactions among all agents in a system, this is referred to as a long-ranged case. Other interaction types are then classified as middle-ranged ones.

2. Multi-clearance statistics

2.1. Empirical multi-clearances

Data record processed in this article has been divided into sub-samples $T_j = \{t_{j1}, t_{j2}, \ldots, t_{jM}\}$ containing M consecutive netto-time intervals between succeeding cars passing a traffic detector located at a chosen lane, where M is the fixed sampling size. For each sub-sample there were calculated the local flux J_j and local density ϱ_j. The method used for those calculations is identical to the standard method described mathematically in Krbalek (2013). In order to avoid an undesirable mixing between different traffic constellations we apply (similarly to approaches presented in Helbing (2004); Krbalek (2009, 2013)) the procedure commonly called (in mathematical disciplines) as *unfolding*. For purposes of our research, such a procedure is composed of two components: a *re-scaling* and *segmentation* by a flux-density window. To be specific, for a certain window-size $(\Delta_J, \Delta_\varrho)$ we introduce a flux-density window $W(\varrho, J) := [\varrho, \varrho + \Delta_\varrho] \times [J, J + \Delta_J]$ that represents a small rectangular sub-region inside the flux-density map. Then the re-scaling is understood as a transformation of the set T_j into the set $\{\tau_{j1}, \tau_{j2}, \ldots, \tau_{jM}\}$, where $\tau_{jk} = t_{jk} / \sum_{k=1}^{M} t_{jk}$. Furthermore, the segmentation

$$I(\varrho, J) = \{j : J_j \in [J, J + \Delta_J] \wedge \varrho_j \in [\varrho, \varrho + \Delta_\varrho]\} \tag{1}$$

is understood as a procedure selecting all sub-samples associated with a chosen flux-density window $W(\varrho, J)$. It means that the final sample of unfolded clearances looks like

$$\mathcal{T}(\varrho, J) = \{\tau_{jk} : j \in I(\varrho, J) \wedge k = 1, 2, \ldots, M\}. \tag{2}$$

Elements of $\mathcal{T}(\varrho, J)$ are here referred to as *unfolded time-clearances* or simply *time-clearances.* Introducing now *multi-clearances of order n* by a definition

$$\tau_{jk}(n) = \tau_{jk} + \tau_{j(k+1)} + \ldots + \tau_{j(k+n)}, \quad j \in I(\varrho, J) \wedge k = 1, 2, \ldots, M - n$$

we calculate an empirical histogram $H(\tau|n)$ quantifying statistical distributions of time-gaps among $n + 2$ succeeding cars in moving clusters of vehicles (associated with fixed values of the flux and density).

Nomenclature

τ	time-clearance between two cars (mathematically interpreted)
M	the sampling size
j	index of a sub-sample
k	index of a car in a sub-sample
τ_{jk}	empirical time-clearance among immediately neighboring vehicles
J_j	the local flux of a sub-sample
ϱ_j	the local density of a sub-sample
W	the flux-density window
$\tau_{jk}(n)$	empirical multi-clearance of order n
$\wp(\tau)$	theoretical probability density for time-clearance among immediately neighboring vehicles
$\wp(\tau\|n)$	theoretical probability density for time-clearance among $n + 2$ neighboring vehicles
$H(\tau)$	empirical histogram of time-clearances among immediately neighboring vehicles
$H(\tau\|n)$	empirical histogram of time-clearances among $n + 2$ neighboring vehicles