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An inverse geometric problem: Position and shape identification of inclusions in a conductor domain ☆

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ABSTRACT

This work presents a methodology for identifying inclusions in a conductor domain. The methodology is based on electrical potential measurements on the external boundary of a conductor body subjected to a prescribed set of electrical current injections. The boundary of each inclusion is approximated by a special kind of spline, whose control points have an extra parameter related to the distance between the control point and the curve. Such special feature allows identification of smooth or sharp inclusions with the same initial guess. The identification is an inverse problem that, in this work, is solved by the Levenberg–Marquardt Method. This iterative method tries to locate the minimum of an objective function, the square of the norm of a residual vector function, given by the differences between electrical potential measurements and the computed ones. The computation of the electrical potential is called forward problem and it is solved by an implementation of the direct formulation of the Boundary Element Method (BEM). The present paper addresses two approaches for computing the derivatives of the residual function with respect to the minimization parameters, required by the Levenberg–Marquardt Method. The first one is based on finite differences and the second one is based on the direct differentiation of the integral equation of the BEM for potential problems. Performance comparisons of these two approaches are presented, based on numerical experiments of identification of inclusions with noisy datasets computationally obtained.

1. Introduction

The problem treated in this paper consists in identification of an inclusion of known conductivity in a homogeneous body. This process is based on electrical potentials measures on the external boundary of the body, generated by a predefined set of electrical current injections. This inverse problem is related to the Electrical Impedance Tomography (EIT) [1,2], which is applied in a large variety of situations like industrial process control [3,2] and medical applications. In this latter area of application the technique can be used for tumors identification [4], diagnostic of breast carcinoma [5,6], mapping of cerebral activity [7], lung ventilation [8] and cardiac function monitoring [9,10]. The special interest of using the EIT in medical applications is due to its radiation-free property, allowing this technique to be used continuously. Other advantages are its portability and low costs compared with other common techniques, such as magnetic resonance and computed tomography. On the other hand, its spacial resolution is lower than other imaging methods.

In this paper, the inverse problem of reconstructing the conductivity distribution in a 2D domain can be viewed as a data fitting problem. The aim of the methodology is to find the vector of geometric parameters that minimizes the misfit between computed and measured boundary potentials. These parameters are positions of control points of splines describing the boundary of the inclusion to be determined as shown in Section 3. With this strategy, the problem can be mathematically described as follows:

$$\chi^* = \underset{\chi}{\operatorname{argmin}}\{F(\chi)\}, \tag{1}$$

$$F(\chi) = \tfrac{1}{2}\mathbf{f}(\chi)^T\mathbf{f}(\chi), \tag{2}$$

$$\mathbf{f}(\chi) = \mathbf{z}(\chi) - \overline{\mathbf{z}}, \tag{3}$$

where $\chi \in \mathbb{R}^n$ is the vector of optimization parameters, $F(\chi) : \mathbb{R}^n \to \mathbb{R}$ is the objective function, $\mathbf{f} : \mathbb{R}^n \to \mathbb{R}^m$ is the residual vector function, $\mathbf{z} : \mathbb{R}^n \to \mathbb{R}^m$ is the vector of computed potential values, $\overline{\mathbf{z}} \in \mathbb{R}^m$ is the vector of measured potentials, m is the number of potential

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measures and n is the number of optimization variables. As $\mathbf{z}$ is a nonlinear function of χ, Eqs. (1)–(3) define a nonlinear least-square problem.

In order to solve the optimization problem presented above, the well known method Levenberg–Marquardt, described for instance in [11], was chosen. This method is considered one of the best to solve this kind of problems [12,13] and requires the computation of derivatives of the objective function with respect to the optimization parameters. The information provided by these derivatives is used to improve the efficiency of the method, reducing number of objective function evaluations, when compared with zero order methods.

In the present paper, two approaches for computing the derivatives are addressed. The first is the usual based on finite differences and the second one is based on the direct differentiation of the integral equation of the BEM for potential problems.

Next section presents the Governing Equation of the forward problem, used to compute the boundary potential values for a given conductivity distribution. Then, Section 3 defines the parameterization used and Section 4 briefly describes the algorithm used to solve the inverse problem. Both approaches for computing the derivatives are presented in Section 5. In Section 6, numerical experiments for identification of inclusions are presented. In this section, the performance of the two approaches for computing derivatives is compared and the influence of noisy datasets is assessed. Finally, discussions and conclusions about the methodology are presented in Section 7.

2. Governing equation

As usual, the inverse problem here defined is solved iteratively. At each iteration the vector χ is updated, demanding a new solution of the forward problem in order to obtain the potentials at the boundary needed for the evaluation of the objective function.

Although electromagnetic phenomena are governed by Maxwell's equations, in the problem here treated it is assumed that the frequency is maintained low, hence the problem can be conveniently approximated by Laplace's equation [14].

Here, we consider a domain composed by a main conductor material with inclusions of different conductivities such that it can be divided in homogeneous subdomains as shown in (Fig. 1). The potential value u at point $\mathbf{x}$ of the k-th subdomain must satisfy

$$\nabla^2 u_k(\mathbf{x}) = 0, \quad \mathbf{x}\in\Omega_k, \tag{4}$$

where Ω_k represents the k-th subdomain. The main subregion is represented by Ω_0 and the inclusions are represented by Ω_k, with $k>0$.

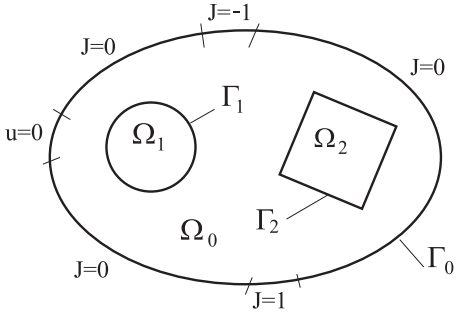

Fig. 1. An example of heterogeneous domain divided in three homogeneous subdomains. A scheme of the boundary conditions of the Laplace's problem is presented.

The boundary and the interface conditions are

$$u_0(\mathbf{x}) = 0, \quad \mathbf{x}\in\Gamma_0^U; \tag{5}$$

$$\sigma_0 \frac{\partial u_0}{\partial \mathbf{n}} = J(\mathbf{x}), \quad \mathbf{x}\in\Gamma_0^J; \tag{6}$$

$$u_0(\mathbf{x}) = u_k(\mathbf{x}), \quad \mathbf{x}\in\Gamma_k; \tag{7}$$

$$\sigma_0 \frac{\partial u_0}{\partial \mathbf{n}}(\mathbf{x}) = -\sigma_k \frac{\partial u_k}{\partial \mathbf{n}}(\mathbf{x}), \quad \mathbf{x}\in\Gamma_k; \tag{8}$$

where σ_k is the conductivity of k-th subdomain, $\mathbf{n}$ is the outward normal to the boundaries of Ω_0, $\partial u_k/\partial \mathbf{n} = \nabla u_k \cdot \mathbf{n}$ is the normal derivative of the potential, Γ_0 is the external boundary of the domain, Γ_k is the interface between the subdomains Ω_0 and Ω_k and $J(\mathbf{x})$ is the current density prescribed.

For each current injection case, two electrodes (Γ_0^J) are used, one to inject and the other to drain electrical current. Here we adopt $J(\mathbf{x})=1$ and $J(\mathbf{x})=-1$ at these electrodes. A third electrode (Γ_0^U), used as potential reference, has null potential prescribed. The rest of the external boundary (Γ_0^J) has null current density prescribed.

The Boundary Element Method (BEM) was chosen to solve the above described forward problem mainly due to the simplicity in mesh generation, that is demanded at each iteration. Additionally, there is no need of computation of internal unknowns, since the objective function depends only on the potential values at the external boundary of the body.

BEM is nowadays a standard numerical method for the solution of the kind of problem here treated and details of its derivation can be found elsewhere [15]. In order to provide details of the present implementation a brief summary is presented below. The adopted formulation is based on the following integral equation:

$$c(\xi)u(\xi) + \int_\Gamma p^*(\xi;\mathbf{x})u(\xi;\mathbf{x})\, d\Gamma(\mathbf{x}) = \int_\Gamma u^*(\xi;\mathbf{x})p(\xi;\mathbf{x})\, d\Gamma(\mathbf{x}), \tag{9}$$

where ξ is the collocation point, Γ is the boundary of the subdomain, u is the electrical potential, p is its normal derivative, u^* and p^* are the fundamental solutions for the potential and its normal derivative, respectively and $c(\xi)$ is a function of the boundary shape, whose value is 0 if ξ is outside of the domain, 1 if $\xi\in\Omega$ and $\beta/2\pi$ if $\xi\in\Gamma$. The parameter β is the angle between the left and right tangents at the collocation point ξ.

In order to obtain a numerical solution for Eq. (9), the boundary of the body is discretized. The external boundary is divided in N_0 elements and each inclusion boundary in N_k elements. The element here adopted has a linear geometry and the values of the electrical potential and its normal derivative are constants along each element. Therefore, each boundary element has two nodes for geometric definition and a centered node for the potential and flux approximation. In this case, the parameter $\beta=\pi$ and then $c(\xi)=0.5$ if $\xi\in\Gamma$. Thus, the discretized form of Eq. (9) for each subdomain k allows evaluating the potential at each functional node as follows:

$$c(\xi_i)u(\xi_i) + \sum_{j=1}^{N_k} u_j \int_{\Gamma_j} p^*\, d\Gamma_J = \sum_{j=1}^{N_k} p_j \int_{\Gamma_j} u^*\, d\Gamma_J, \tag{10}$$

where u_j and p_j represent the potential and its derivative at the j-th node, the regular integrals are computed numerically by the usual Gauss Quadrature scheme and the singular ones are computed analytically.

The application of Eq. (10) for each subdomain Ω_k, in addition to the boundary, Eqs. (5) and (6), and the compatibility conditions, Eqs. (7) and (8) for the potential and flux at the functional nodes of the interface elements at Γ_k, yields a linear system of algebraic