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A geometric inverse problem identification procedure for detection of cavities ☆

L.P.S. Barra [a,*], J.C.F. Telles [b]

[a] Federal University of Juiz de Fora, Campus Universitário, CEP 36036-330 Juiz de Fora, MG, Brazil
[b] Department of Civil Engineering, COPPE/UFRJ, Caixa Postal 68506, CEP 21941-972 Rio de Janeiro, RJ, Brazil

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ABSTRACT

In this paper the inverse problem of electrical impedance tomography (EIT) in a three dimensional environment is considered. In this technique, electrodes are placed on the external boundary of the body and electrical current is injected by sequentially activating pairs of them while the corresponding potentials are measured. Usually such measures are used in order to solve the nonlinear inverse problem of achieving a two-dimensional image of the conductivity distribution over the cross section of the body. In the problem studied here the goal is to determine the size and position of an existing cavity within a homogeneous medium. The geometrical parameters that describe the cavities are the unknowns of the resulting 3D inverse problem, which is solved by the Levenberg–Marquardt method. Two shapes of geometrical cavities are here considered: spherical and spheroidal. Due to its accuracy and simplicity of mesh generation, the Boundary Element Method (BEM) is used in the solution of the direct problem. In order to evaluate the proposed strategy, numerical experiments are presented varying the position and the shape of the cavity and also the injection-measure protocol used. Since measured data are not currently available, boundary potential measurements have been obtained computationally also using BEM. The sensitivity of the present method in the presence of measurement noise has also been estimated through numerical experiments.

1. Introduction

The present paper addresses the problem of identification of a cavity, in a conductive domain, starting from the measurements of electrical potential on electrodes placed over the external boundary of the body. The problem is dealt with by first finding the required geometrical parameters of the cavity to be identified and then trying to find their actual values. This is done by the minimization of the misfit between the electrical potential boundary measurements and the corresponding values obtained with the numerical model used. These three-dimensional potential distributions are generated by a series of current injections through electrodes placed on the boundary of the body, on the 3-D domain Ω. The numerical values of the electrical potential u are obtained by solving the Laplace equation

$$\nabla^2 u = 0 \tag{1}$$

using a direct formulation of the Boundary Element Method (BEM). This leads to a nonlinear least-squares problem, with the geometrical parameters as unknowns.

This kind of problem can arise, for instance, in monitoring flow fields in industrial processes [1].

Another problem related to the one here investigated comes from the electrical impedance tomography (EIT) [2]. In the EIT inverse problem, based on the same kind of data, the aim is to compute the distribution of electrical conductivity within a domain. EIT is used in a wide range of applications. In medical imaging this technique is used, for instance, to monitor pulmonar function [3] and is a promise for detecting and characterizing tumors in the breast [4]. In geophysical community this technique is known as electrical resistivity tomography (ERT) and is a common tool for aquifer characterization [5,6]. Recently, an experimental study [7] has shown that EIT can become a feasible modality for non-destructive evaluation of concrete. Numerical solutions of this problem usually decompose the domain in small sub-domains whose conductivities are the unknowns of the inverse problem. Within this approach the Finite Element Method is the usual choice for the direct problem solution. In this case the

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* Corresponding author. Tel.: +55 32 8407 0305; fax: +55 32 2102 3401.
E-mail addresses: luis.barra@ufjf.edu.br (L.P.S. Barra), telles@coc.ufrj.br (J.C.F. Telles).

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number of unknowns is much larger than the number of restrictions (measurements) leading to a rank-deficient ill-posed problem. Hence, the approach demands the usage of regularization methods to find acceptable solutions.

If there is some knowledge of the structure of the searched conductivity distribution, a similar approach to the strategy used here can be used in the EIT problem. In the case of sub-domains with different conductivities, geometrical parameterizations can be used to define internal boundaries. This can be viewed as an introduction of *a priori* information in order to regularize the problem and significantly reduces the number of unknowns of the inverse problem. This kind of methodology has already been applied in a two-dimensional numerical setup to the problem of heart function monitoring [8].

In order to assess the limitations of the proposed approach for 3-D problems, the application of one cavity identification is studied in the present work. Two kinds of shape parameterizations are adopted for the cavities, respectively with four and seven parameters: spherical and spheroidal.

Also two different protocols of current injection are compared when used to solve the problem of identifying a cavity in a set of different locations within the domain. The numerical results obtained are compared in order to identify the most adequate. In order to solve the nonlinear least-squares problem two strategies are compared. The first one is a classical routine of the Levenberg–Marquardt method and the second is a generalized secant version of this algorithm. In this kind of approach here adopted, the evaluation of the function to be minimized is the most time consuming part of the solution and this second version aims to reduce the number of evaluations needed to achieve the solution.

As there were no real boundary potential data available from experiments, numerical generated data is used instead. In order to assess the influence of noise in the measurements, disturbances in the synthetic data are added and their influence in the numerical inversion results are reported.

2. Inverse problem definition

The aim of the present strategy is to find the vector containing these geometrical parameters $\mathbf{t}^*$ corresponding to the model results that best fit the measured data. Mathematically this can be written as the minimization of a function f:

$$f = \frac{1}{2}\mathbf{R}(\mathbf{t})^T\mathbf{R}(\mathbf{t}) \tag{2}$$

with

$$\mathbf{R}(\mathbf{t}) = \mathbf{V}(\mathbf{t}) - \overline{\mathbf{V}} \tag{3}$$

where f is the objective function ($f : \mathbb{R}^n \rightarrow \mathbb{R}$), $\mathbf{R}(\mathbf{t})$ is the residual function ($\mathbf{R} : \mathbb{R}^n \rightarrow \mathbb{R}^m$), $\overline{\mathbf{V}}$ is a vector with the m electrical potential measures and $\mathbf{V}(\mathbf{t})$ stores the computed potentials when a set of n geometrical parameters are assigned.

The number of possible independent measures that can be used to study the problem depends on the number of electrodes used in the idealized experimental setup and on the current injection pattern used to induce the potential measures. Although, in principle, a general pattern of current injection could be used, due to hardware simplicity, most of the EIT equipments use a single source for current injection [2]. This implies that only two electrodes are used in each injection complemented by another one for potential reference. Also, the utilization of 16 electrodes is a common practice in EIT procedures, hence this was the number of electrodes assumed in the present work. These are numbered from 1 to 8 in the lower plane and from 9 to 16 in the upper plane as depicted in Fig. 1.

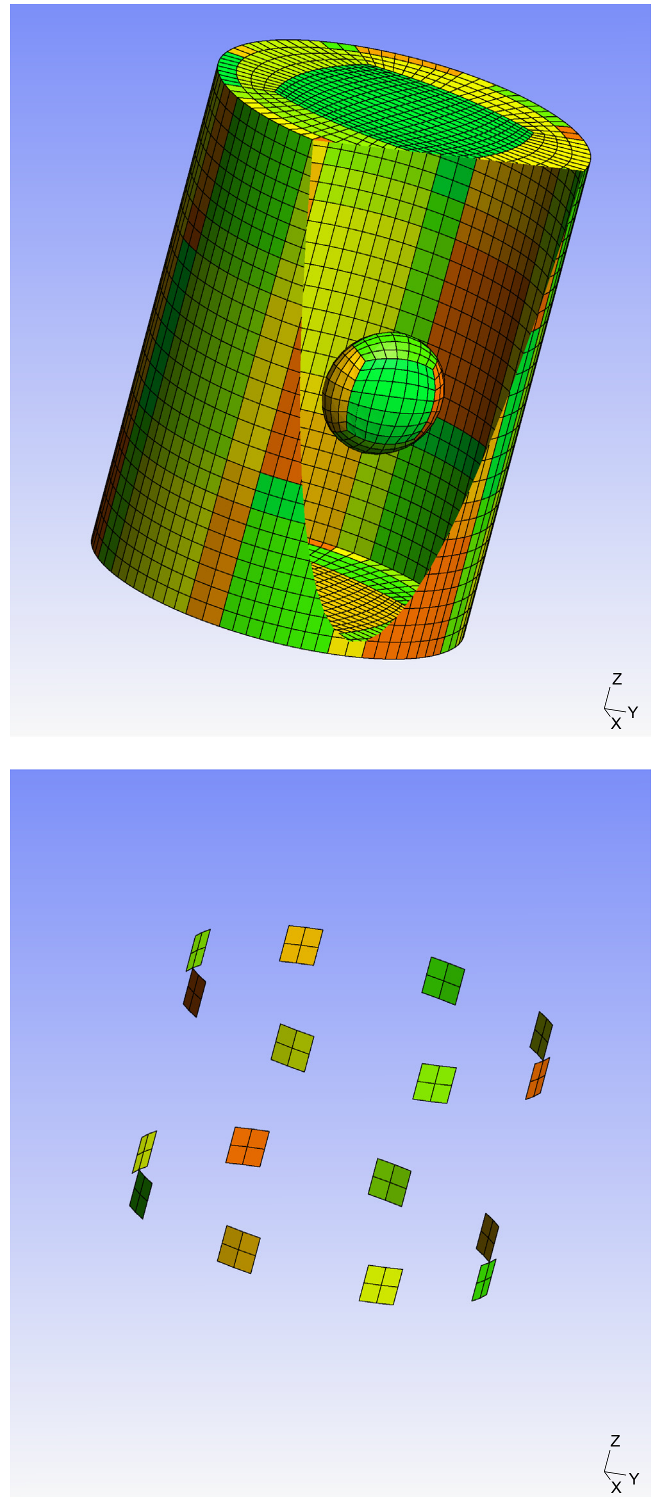

Fig. 1. Mesh of the exterior and interior boundaries (above); modelled electrodes for current injection and potential measurements.

In two dimensional problems two patterns are commonly used [9]: the adjacent and the opposite patterns, where two consecutive and diametrically opposed electrodes are respectively used for the current injections. In the three dimensional case the definition of the more appropriate pattern is still under investigation [10,11]. The two patterns described in Table 1 have been used in the present work. The first one is composed of three sets of injections in accordance with the opposite pattern. In the first set, the four