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# Stress distribution of mine roof with the boundary element method

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ABSTRACT

Mine roof, is a stiff rock strata, located on the top of coal seam, which can prevent the deformation and control the stability of coal roadway after the coal roadway is tunneled, so mine roof is one of the most important structures in coal mining engineering. In this paper, mine roof is treated as elastic plate, which is studied thoroughly at the theoretical level. Based on the mechanical models of plane and stress analysis for elastic roof, using the boundary integral equation which is obtained by the natural boundary reduction, this paper obtains stress functions of elastic half roof, as well as the analytical and numerical solutions to the each stress field functions. We also analyze the rules of different stress distributions for roof under a concentrated force and a uniform distribution load, the results of calculation show uniformity of the stress distribution. In order to research the mine roof deformation law, Mohr–Coulomb model is established to describe the deformation behavior of roof surrounding rock, $FLAC^{3D}$ is also used to simulate the deformation of roof after the coal roadway is tunneled under different length of coal roadway excavation. The comparison result between BEM solution and $FLAC^{3D}$ simulation shows advantages to solve the problem by boundary element method, and numerical simulation proves the deformation behavior of roof is influenced by the length of coal roadway excavation.



## 1. Introduction

Mine roof plays an important role in coal mining engineering especially in longwall mining, it can prevent the deformation and control the stability of coal roadway after the coal roadway is tunneled. However, the fracture of the mine roof would affect the distribution of the abutment pressure ahead of the longwall face and a series of phenomena would happen around the face area. For instance, the convergence in working area and load acting on the support would be increased, and sometimes some kinetic phenomena might happen [18].

In generally, rock strata are usually regarded as plate in the working area. Miao et al. [18] treated the rock strata as elastic plate. For the elastic plate, the stress solution in different engineering problems have been analyzed by various methods [21,13,11,22,9,1,2,20,3,4,12], so far, Ma and Mao [15] and Ma et al. [16] have analyzed the stress distribution of elastic roof and water resisting key strata. However, the elastic half roof problem in coal mining engineering, such as the stress analysis of roof under different coal roadway excavation, is still not solved so far, and it needs to be studied thoroughly at the theoretical level.

The boundary element method (BEM) [8] solves field problems by solving an equivalent source problem. In the case of electric fields, it solves for equivalent charge, while in case of magnetic fields, it solves for equivalent currents. BEM also uses an integral formulation of Maxwell's Equations, which allows for accurate field calculations. The elastic problems also be solved by BEM well [6,7].

In theory, it is convenient to analyze the problems of elastic half roof under various boundary conditions using the natural BEM. The natural BEM [25] is a branch of a number of BEMs based on a complex variable method, a method using a Fourier series, or a Green's function method to induce a Dirichlet boundary value problem as a differential equation into Poisson integration equation of the studied area or to induce Neumann boundary value problem of differential equation into a strong singular boundary integral equation [25]. The natural BEM is widely used to solve problems of a circular interior and exterior domain plane and engineering problems, as rock surrounding the roadway [17], elastic roof and water resisting key strata [16].

This paper uses the surface forces on the boundary to calculate the stress function and its normal derivative, which are substituted into the integration equation, then obtains the specific expression of a stress function under various boundary conditions, and permits of the analysis of stress and related deformation for the elastic half roof, and then the rules of stress distribution varying with $x$ and $z$ are analyzed. In order to research the deformation

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law of roof under different length of coal roadway excavation, Mohr–Coulomb model and FLAC$^{3D}$ are used to simulate the deformation behavior of roof after the coal roadway is tunneled.

## 2. Structure model of mine roof

In general, there exist the obvious characteristics of layers of coal seam rock strata, such as some solid strata have great ability of loading, and for other strata, which contain mud or clay minerals with a good anti-seepage ability, have the functions of water-preserved. Fig. 1 shows the structure model of roof, the rock strata, coal seam, floor strata and mine roof, can be regarded as elastic rectangle plate for the following analyses.

## 3. Poisson integral bi-harmonic equation under natural boundary reduction

The roof can be regarded as plate for the following analyses. Assume the boundary value problem of the bi-harmonic equation is as follows:

$$\begin{cases} \Delta^2\phi = 0, & \text{in } \Omega \\ \phi|_\Gamma = \phi_0, & \dfrac{\partial\phi}{\partial n}|_\Gamma = \phi_n \end{cases} \tag{1}$$

where $\phi_0$ and $\phi_n$ are the stress function on the boundary $\Gamma$ and its normal derivative respectively. Suppose differential boundary value operator

$$\begin{cases} T\phi = (-\frac{\partial}{\partial n}\Delta\phi)|_\Gamma \\ M\phi = (\Delta\phi)|_\Gamma \end{cases} \tag{2}$$

where, $M$ and $T$ are the differential boundary operator respectively.

According to the second Green formula of bi-harmonic equation [25],

$$\iint_\Omega (\phi\Delta^2\phi - \phi\Delta^2\varphi)\mathrm{d}\,p = \int_\Gamma \left(\phi\frac{\partial}{\partial n}\Delta\phi - \frac{\partial\phi}{\partial n}\Delta\phi + \frac{\partial\phi}{\partial n}\Delta\phi - \phi\frac{\partial}{\partial n}\Delta\phi\right)\mathrm{d}\,s \tag{3}$$

where $\mathrm{d}\,p = \mathrm{d}\,x\mathrm{d}\,z$, if $\phi = \phi(p)$ satisfies the bi-harmonic equation (Eq. (1)), and $\varphi = G(p,p')$ is Green function of bi-harmonic equation within $\Omega$, namely

$$\begin{cases} \Delta^2 G(p,p') = \delta(p-p') \\ G(p,p')|_\Gamma = 0 \\ \frac{\partial}{\partial n}G(p,p')|_\Gamma = 0 \end{cases} \tag{4}$$

then the Poisson integral bi-harmonic equation [25] can be obtained as

$$\phi(p) = \int_\Gamma [-T'G(p,p')\phi_0(p') - M'G(p,p')\phi_n(p')]\mathrm{d}\,s', \quad p \in \Omega \tag{5}$$

where $p=(x,z)$, $p'=(x',z')$, $M'$ and $T'$ are the differential boundary value operators about $(x',z')$. Eq. (5) can also be rewritten as

$$\phi(p) = \int_\Gamma \left[\frac{\partial}{\partial n'}\Delta' G(p,p')\phi_0(p') - \Delta' G(p,p')\phi_n(p')\right]\mathrm{d}\,s', \quad p \in \Omega \tag{6}$$

Eq. (6) is the Poisson integral bi-harmonic equation in general domain under the natural boundary reduction.

## 4. Boundary element formulations for elastic half roof

Suppose the boundary $\Gamma$ of the half roof domain $\Omega$ is $x$ axis (as shown in Fig. 2), and the exterior normal derivative on boundary $\Gamma$ is $(\partial/\partial n) = -(\partial/\partial z)$. From fundamental solution of bi-harmonic equation, Green function $G(p,p')$ [25] of bi-harmonic equation of the elastic half roof problem is

$$G(p,p') = \frac{1}{16\pi}[(x-x')^2+(z-z')^2] \times \ln\frac{(x-x')^2+(z-z')^2}{(x-x')^2+(z+z')^2} + \frac{zz'}{4\pi} \tag{7}$$

From Eq. (7), we obtain

$$\Delta' G|_\Gamma = \Delta' G|_{y'=0} = \frac{z^2}{\pi[(x-x')^2+z^2]} \tag{8}$$

$$\frac{\partial}{\partial n'}\Delta' G\bigg|_{y'=0} = \frac{2z^3}{\pi[(x-x')^2+z^2]^2} \tag{9}$$

Substituting Eqs. (7)–(9) into (6), we can obtain the Poisson integral formula of stress function for the bi-harmonic equation of the half roof body problem

$$\phi(x,z) = \frac{1}{\pi}\int_{-\infty}^{\infty} \frac{2z^3}{[(x-x')^2+z^2]^2}\phi_0(x')\mathrm{d}\,x'$$

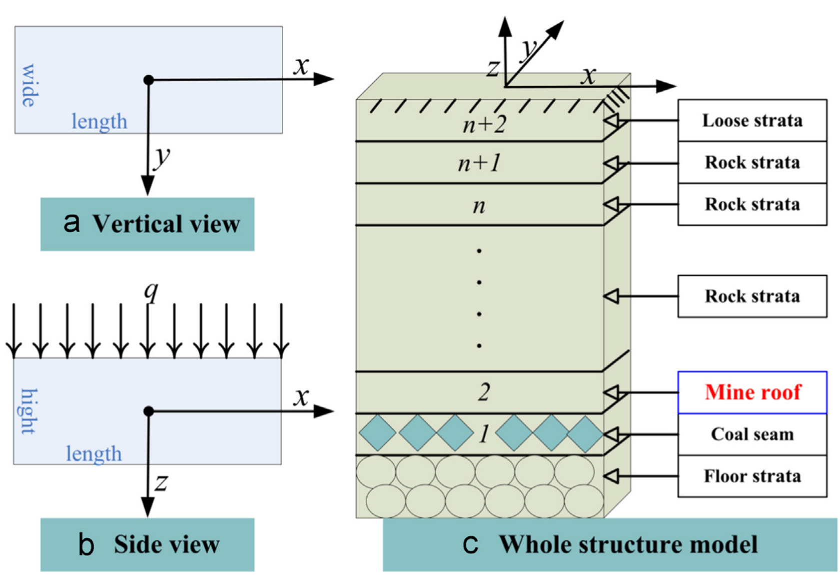


**Fig. 1.** Structure model of mine roof in coal mining engineering.